25. Parabola

Exercise 25.1

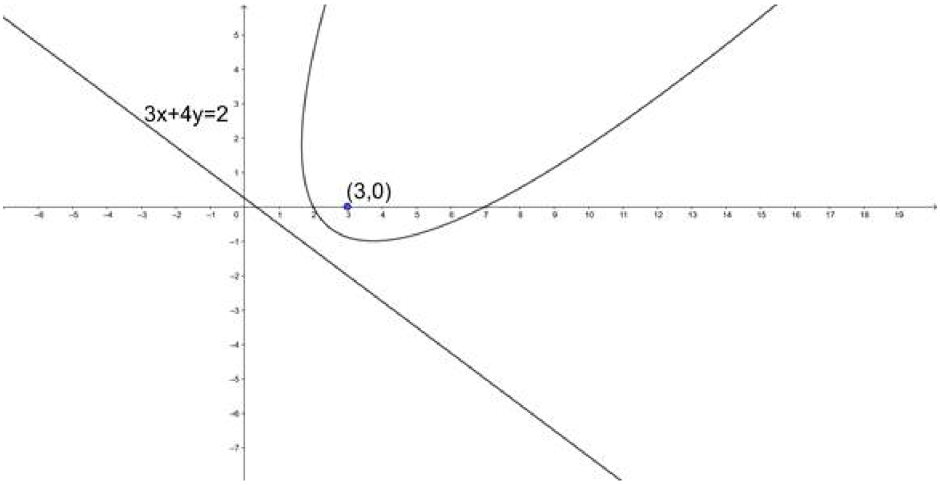
1 A. Question

Find the equation of the parabola whose:

focus is (3, 0) and the directrix is 3x + 4y = 1

Answer

Given that we need to find the equation of the parabola whose focus is S(3, 0) and directrix(M) is 3x + 4y - 1 = 0.



Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 3)^2 + (y - 0)^2 = \left(\frac{|3x + 4y - 1|}{\sqrt{3^2 + 4^2}}\right)^2$

⇒ $x^2 - 6x + 9 + y^2 = \frac{(|3x + 4y - 1|)^2}{3^2 + 4^2}$

⇒ $x^2 + y^2 - 6x + 9 = \frac{(9x^2 + 16y^2 + 1 - 6x - 8y + 24xy)}{9 + 16}$

⇒ $25x^2 + 25y^2 - 150x + 225 = 9x^2 + 16y^2 - 6x - 8y + 24xy + 1$

⇒ $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$

∴The equation of the parabola is $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$

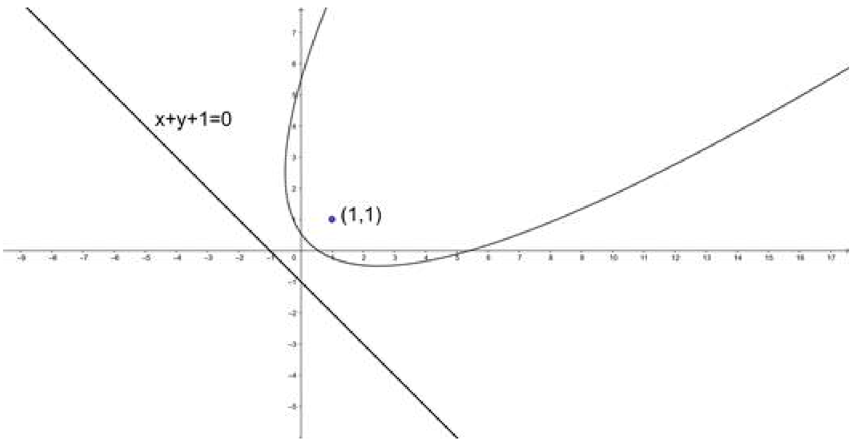
1 B. Question

Find the equation of the parabola whose:

focus is (1, 1) and the directrix is x + y + 1 = 0

Answer

Given that we need to find the equation of the parabola whose focus is S(1, 1) and directrix(M) is x + y + 1 = 0.



Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 1)^2 + (y - 1)^2 = \left(\frac{|x + y + 1|}{\sqrt{1^2 + 1^2}}\right)^2$

⇒ $x^2 - 2x + 1 + y^2 - 2y + 1 = \frac{(|x + y + 1|)^2}{1 + 1}$

⇒ $x^2 + y^2 - 2x - 2y + 2 = \frac{(x^2 + y^2 + 1 + 2x + 2y + 2xy)}{2}$

⇒ $2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 2x + 2y + 2xy + 1$

⇒ $x^2 + y^2 + 2xy - 6x - 6y + 3 = 0$

∴The equation of the parabola is $x^2 + y^2 + 2xy - 6x - 6y + 3 = 0$.

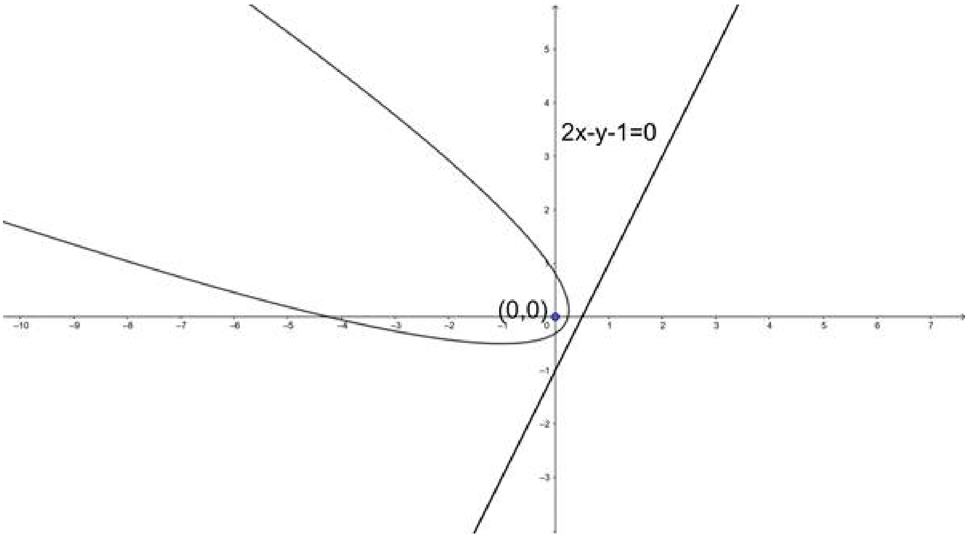
1 C. Question

Find the equation of the parabola whose:

focus is (0, 0) and the directrix is 2x - y – 1 = 0

Answer

Given that we need to find the equation of the parabola whose focus is S(0, 0) and directrix(M) is 2x - y - 1 = 0.



Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 0)^2 + (y - 0)^2 = \left(\frac{|2x - y - 1|}{\sqrt{2^2 + (-1)^2}}\right)^2$

⇒ $x^2 + y^2 = \frac{(|2x - y - 1|)^2}{4 + 1}$

⇒ $x^2 + y^2 = \frac{(4x^2 + y^2 + 1 - 4x + 2y - 4xy)}{5}$

⇒ $5x^2 + 5y^2 = 4x^2 + y^2 - 4x + 2y - 4xy + 1$

⇒ $x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$

∴The equation of the parabola is $x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$.

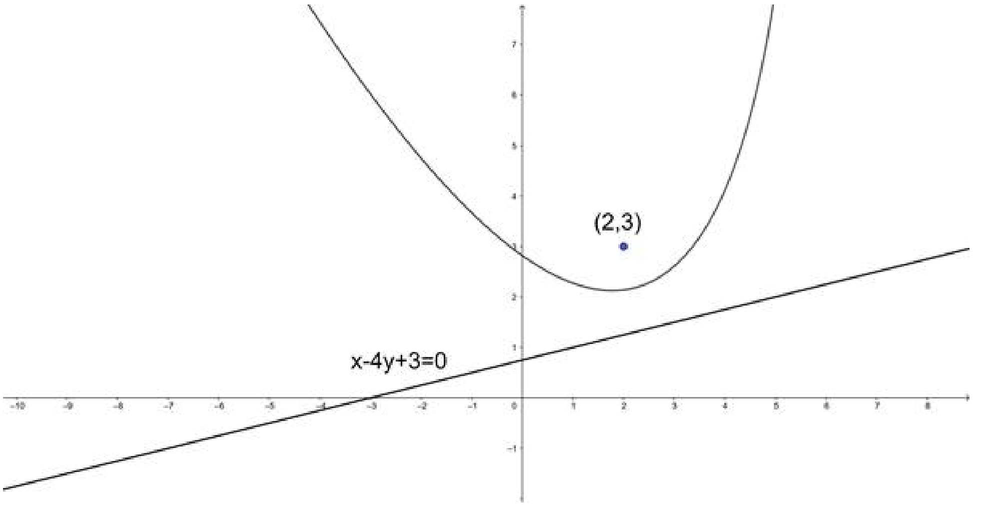
1 D. Question

Find the equation of the parabola whose:

focus is (2, 3) and the directrix is x - 4y + 1 = 0

Answer

Given that we need to find the equation of the parabola whose focus is S(2, 3) and directrix(M) is x - 4y + 3 = 0.



Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x-2)^2 + (y-3)^2 = \left(\frac{|x-4y+3|}{\sqrt{1^2 + (-4)^2}}\right)^2$

⇒ $x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{(|x-4y+3|)^2}{1+16}$

⇒ $x^2 + y^2 - 4x - 6y + 13 = \frac{(x^2 + 16y^2 + 9 + 6x - 24y - 8xy)}{17}$

⇒ $17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 6x - 24y - 8xy + 9$

⇒ $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

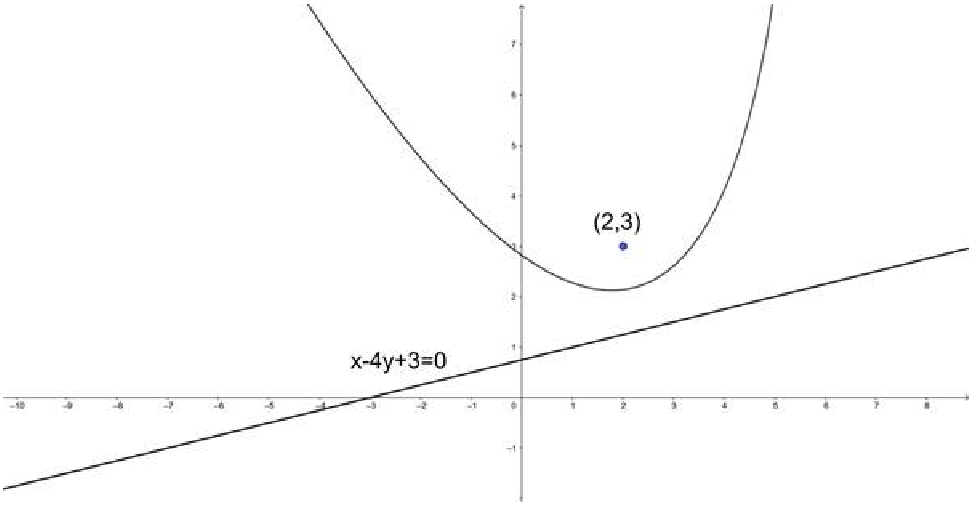
∴The equation of the parabola is $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$.

2. Question

Find the equation of the parabola whose focus is the point (2, 3) and directrix is the line x – 4y + 3 = 0. Also, find the length of its latus - rectum.

Answer

Given that we need to find the equation of the parabola whose focus is S(2, 3) and directrix(M) is x - 4y + 3 = 0.



Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

$\Rightarrow (x-2)^2 + (y-3)^2 = \left(\frac{|x-4y+3|}{\sqrt{1^2+(-4)^2}}\right)^2$

$\Rightarrow x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{(|x-4y+3|)^2}{1+16}$

$\Rightarrow x^2 + y^2 - 4x - 6y + 13 = \frac{(x^2 + 16y^2 + 9 + 6x - 24y - 8xy)}{17}$

⇒ $17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 6x - 24y - 8xy + 9$

⇒ $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

∴The equation of the parabola is $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$.

We know that the length of the latus rectum is twice the perpendicular distance from the focus to the directrix.

$\Rightarrow L = 2\frac{|2-4(3)+3|}{\sqrt{1^2+(-4)^2}}$

$\Rightarrow L = 2\frac{|-7|}{\sqrt{1+16}}$

$\Rightarrow L = \frac{14}{\sqrt{17}}$

∴The length of the latus rectum is $\frac{14}{\sqrt{17}}$.

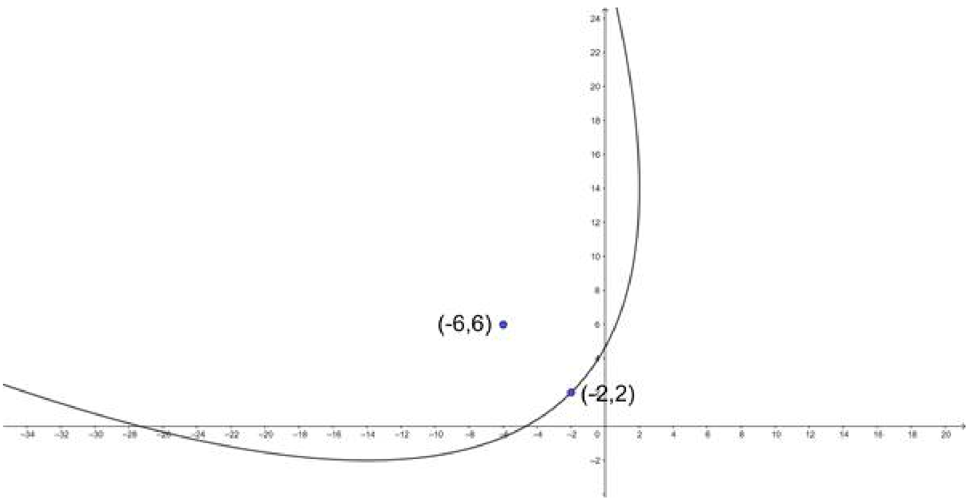
3 A. Question

Find the equation of the parabola, if

the focus is at (- 6, 6) and the vertex is at (- 2, 2)

Answer

We need to find the equation of the parabola whose focus is (- 6, 6), and the vertex is (- 2, 2).



We know that the line passing through focus and vertex, i.e., the axis is perpendicular to the directrix and vertex is the midpoint of focus and point that lies at the intersection of axis and directrix.

⇒ The slope of the axis (m_1) = $\frac{6-2}{-6-(-2)}$

⇒ $m_1 = \frac{4}{-4}$

⇒ $m_1 = -1$

We know that the products of the slopes of the perpendicular lines is - 1.

Let us assume m_2 be the slope of the directrix.

⇒ $m_1.m_2 = -1$

⇒ $-1.m_2 = -1$

⇒ $m_2 = 1$

Let us find the point on directrix.

⇒ $(-2, 2) = \left(\frac{x-6}{2}, \frac{y+6}{2}\right)$

⇒ $\frac{x-6}{2} = -2$ and $\frac{y+6}{2} = 2$

⇒ x - 6 = - 4 and y + 6 = 4

⇒ x = 2 and y = - 2

The point on directrix is (2, - 2).

We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$

⇒ y - (- 2) = 1(x - 2)

⇒ y + 2 = x - 2

⇒ x - y - 4 = 0

Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

$\Rightarrow (x-(-6))^2 + (y-6)^2 = \left(\frac{|x-y-4|}{\sqrt{1^2+(-1)^2}}\right)^2$

$\Rightarrow x^2 + 12x + 36 + y^2 - 12y + 36 = \frac{(|x-y-4|)^2}{1+1}$

$\Rightarrow x^2 + y^2 + 12x - 12y + 72 = \frac{(x^2+y^2+16-8x+8y-2xy)}{2}$

⇒ $2x^2 + 2y^2 + 24x - 24y + 144 = x^2 + y^2 - 8x + 8y - 2xy + 16$

⇒ $x^2 + y^2 + 2xy + 32x - 32y + 128 = 0$

∴The equation of the parabola is $x^2 + y^2 + 2xy + 32x - 32y + 128 = 0$.

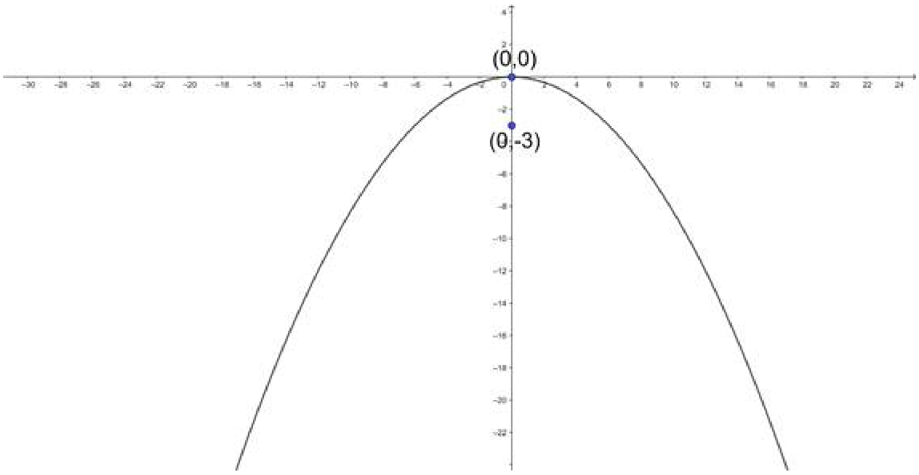
3 B. Question

Find the equation of the parabola, if

the focus is at (0, - 3) and the vertex is at (0, 0)

Answer

We need to find the equation of the parabola whose focus is (0, - 3) and the vertex is (0, 0).



We know that the line passing through focus and vertex i.e., the axis is perpendicular to the directrix and vertex is the midpoint of focus and point that lies at the intersection of axis and directrix.

⇒ Slope of axis $(m_1) = \frac{-3-0}{0-0}$

$\Rightarrow m_1 = \frac{-3}{0}$

⇒ $m_1 = -\infty$

We know that the product of the perpendicular lines is applicable for non - vertical lines.

Here we got the axis to be parallel to the x - axis.

∴The slope of the directrix is equal to the slope of x - axis i.e., 0.

⇒ $m_2 = 0$

Let us find the point on directrix.

$\Rightarrow (0,0) = \left(\frac{x-0}{2}, \frac{y-3}{2}\right)$

$\Rightarrow \frac{x}{2} = 0 \text{ and } \frac{y-3}{2} = 0$

⇒ x = 0 and y - 3 = 0

⇒ x = 0 and y = 3

The point on directrix is (0, 3).

We know that the equation of the lines passing through (x_1, y_1) and having slope m is y - y_1 = m(x - x_1)

⇒ y - 3 = 0(x - 0)

⇒ y - 3 = 0

Let us assume P(x, y) be any point on the parabola.

We know that the point on parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 0)^2 + (y - (-3))^2 = \left(\frac{|y-3|}{\sqrt{1^2}}\right)^2$

⇒ $x^2 + y^2 + 6y + 9 = \frac{(|y-3|)^2}{1}$

⇒ $x^2 + y^2 + 6y + 9 = y^2 - 6y + 9$

⇒ $x^2 + 12y = 0$

∴The equation of the parabola is $x^2 + 12y = 0$.

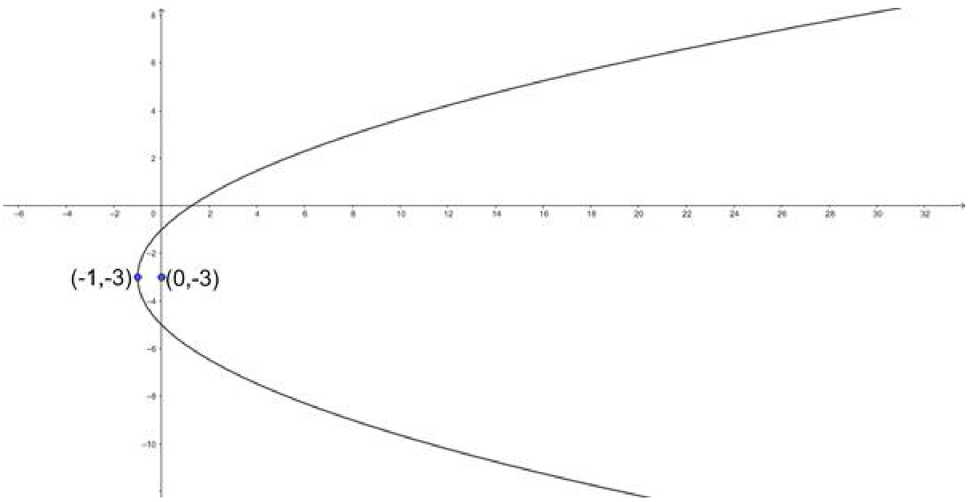
3 C. Question

Find the equation of the parabola, if

the focus is at (0, - 3) and the vertex is at (- 1, - 3)

Answer

We need to find the equation of the parabola whose focus is (0, - 3) and the vertex is (- 1, - 3).



We know that the line passing through focus and vertex i.e., the axis is perpendicular to the directrix and vertex is the midpoint of focus and point that lies at the intersection of axis and directrix.

⇒ Slope of axis (m_1) = $\frac{-3-(-3)}{0-(-1)}$

⇒ $m_1 = \frac{0}{1}$

⇒ $m_1 = 0$

We know that the products of the slopes of the perpendicular lines is - 1 for non - vertical lines.

Here the slope of the axis is equal to the slope of the x - axis. So, the slope of directrix is equal to the slope of y - axis i.e., ∞.

⇒ $m_2 = \infty$

Let us find the point on directrix.

⇒ $(-1, -3) = \left(\frac{x+0}{2}, \frac{y-3}{2}\right)$

⇒ $\frac{x+0}{2} = -1$ and $\frac{y-3}{2} = -3$

⇒ x + 0 = - 2 and y - 3 = - 6

⇒ x = - 2 and y = - 3

The point on directrix is (- 2, - 3).

We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$

⇒ y - (- 3) = ∞(x - (- 2))

⇒ $\frac{y+3}{\infty} = x + 2$

⇒ x + 2 = 0

Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x-0)^2 + (y-(-3))^2 = \left(\frac{|x+2|}{\sqrt{1^2}}\right)^2$

⇒ $x^2 + y^2 + 6y + 9 = \frac{(|x+2|)^2}{1}$

⇒ $x^2 + y^2 + 6y + 9 = x^2 + 4x + 4$

⇒ $y^2 - 4x + 6y + 5 = 0$

∴The equation of the parabola is $y^2 - 4x + 6y + 5 = 0$.

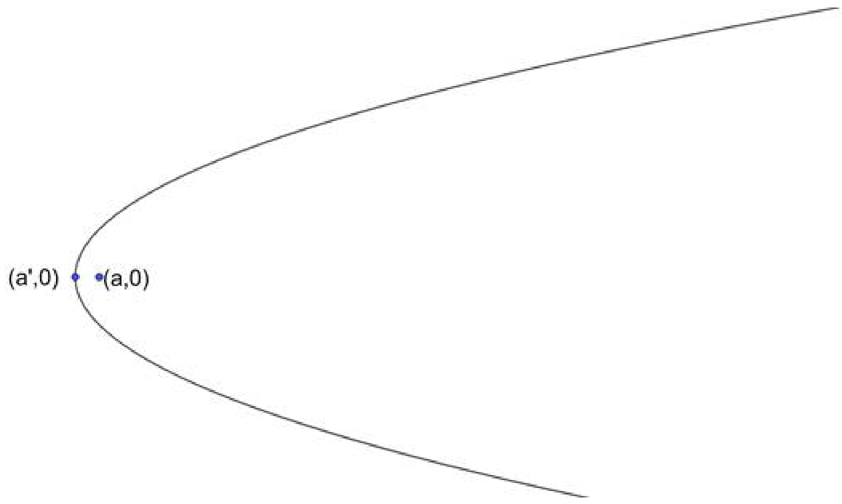
3 D. Question

Find the equation of the parabola, if

the focus is at (a, 0) and the vertex is at (a’, 0)

Answer

We need to find the equation of the parabola whose focus is (a, 0) and the vertex is (a’, 0).



We know that the line passing through focus and vertex i.e., the axis is perpendicular to the directrix and vertex is the midpoint of focus and point that lies at the intersection of axis and directrix.

⇒ Slope of axis $(m_1) = \frac{0-0}{a'-a}$

$\Rightarrow m_1 = \frac{0}{a'-a}$

$\Rightarrow m_1 = 0$

We know that the products of the slopes of the perpendicular lines is - 1 for non - vertical lines.

Here the slope of the axis is equal to the slope of the x - axis. So, the slope of directrix is equal to the slope of y - axis i.e., ∞.

$\Rightarrow m_2 = \infty$

Let us find the point on directrix.

$\Rightarrow (a', 0) = \left(\frac{x+a}{2}, \frac{y+0}{2}\right)$

$\Rightarrow \frac{x+a}{2} = a' \text{ and } \frac{y+0}{2} = 0$

⇒ x + a = 2a’ and y = 0

⇒ x = 2a’ - a and y = 0

The point on directrix is (2a’ - a, 0).

We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$

⇒ y - (0) = ∞(x - (2a’ - a))

$\Rightarrow \frac{y}{\infty} = x + a - 2a'$

⇒ x + a - 2a’ = 0

Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x-a)^2 + (y-0)^2 = \left(\frac{|x+a-2a'|}{\sqrt{1^2}}\right)^2$

$\Rightarrow x^2 - 2ax + a^2 + y^2 = \frac{(|x + a - 2a\prime|)^2}{1}$

$\Rightarrow x^2 + y^2 - 2ax + a^2 = x^2 + a^2 + 4(a')^2 + 2ax - 4aa' - 4a'x$

$\Rightarrow y^2 - (4a - 4a')x + a^2 - 4(a')^2 + 4aa' = 0$

$\therefore$The equation of the parabola is $y^2 - (4a - 4a')x + a^2 - 4(a')^2 + 4aa' = 0$.

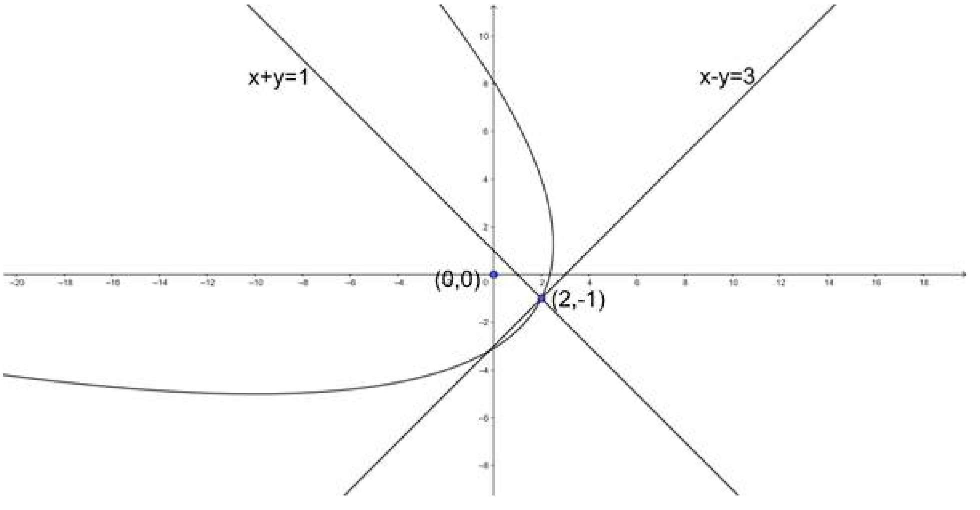
3 E. Question

Find the equation of the parabola, if

the focus is at (0, 0) and vertex is at the intersection of the lines x + y = 1 and x – y = 3

Answer

We need to find the equation of the parabola whose focus is (0, 0) and vertex is the point of intersection of lines x + y = 1 and x - y = 3.



On solving the lines, we get the vertex (2, - 1).

We know that the line passing through focus and vertex i.e., the axis is perpendicular to the directrix and vertex is the midpoint of focus and point that lies at the intersection of axis and directrix.

$\Rightarrow$ Slope of axis $(m_1) = \frac{-1-0}{2-0}$

$\Rightarrow m_1 = \frac{-1}{2}$

We know that the products of the slopes of the perpendicular lines is - 1.

Let us assume m_2 be the slope of the directrix.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow \frac{-1}{2}.m_2 = -1$

$\Rightarrow m_2 = 2$

Let us find the point on directrix.

$\Rightarrow (2, -1) = \left(\frac{x+0}{2}, \frac{y+0}{2}\right)$

$\Rightarrow \frac{x+0}{2} = 2 \text{ and } \frac{y+0}{2} = -1$

$\Rightarrow x = 4$ and $y = -2$

The point on directrix is (4, - 2).

We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$

⇒ $y - (-2) = 2(x - 4)$

⇒ $y + 2 = 2x - 8$

⇒ $2x - y - 10 = 0$

Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 0)^2 + (y - 0)^2 = \left(\frac{|2x - y - 10|}{\sqrt{2^2 + (-1)^2}}\right)^2$

⇒ $x^2 + y^2 = \frac{(|2x - y - 10|)^2}{4 + 1}$

⇒ $x^2 + y^2 = \frac{(4x^2 + y^2 + 100 - 40x + 20y - 4xy)}{5}$

⇒ $5x^2 + 5y^2 = 4x^2 + y^2 - 40x + 20y - 4xy + 100$

⇒ $x^2 + 4y^2 + 4xy + 40x - 20y - 100 = 0$

∴The equation of the parabola is $x^2 + 4y^2 + 4xy + 40x - 20y - 100 = 0$.

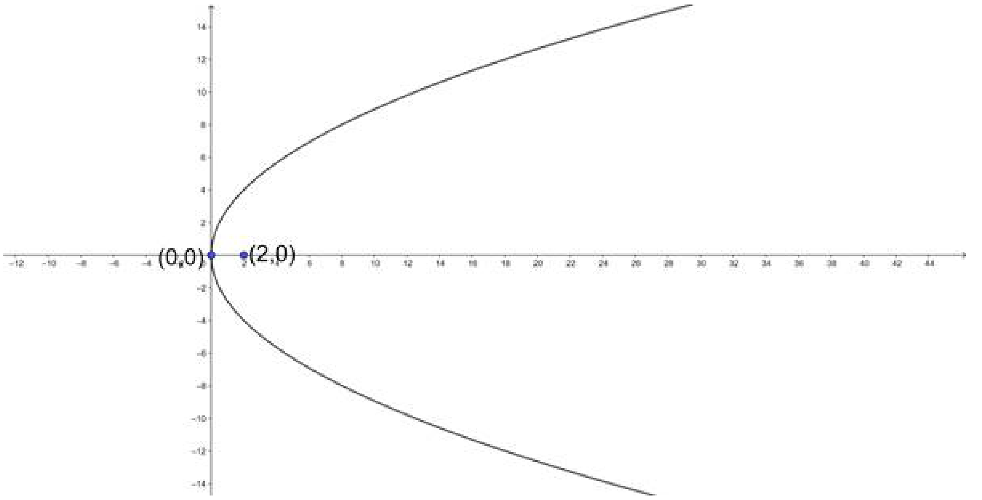
4 A. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$y^2 = 8x$

Answer

Given parabola is $y^2 = 8x$. Comparing this with standard parabola $y^2 = 4ax$ we get,



⇒ 4a = 8

⇒ a = 2

⇒ The vertex is (0, 0)

⇒ The focus is (a, 0) = (2, 0)

⇒ The equation of the axis is y = 0.

⇒ The equation of the directrix is x = - a i.e, x = - 2

⇒ The length of the latus rectum is 4a = 8.

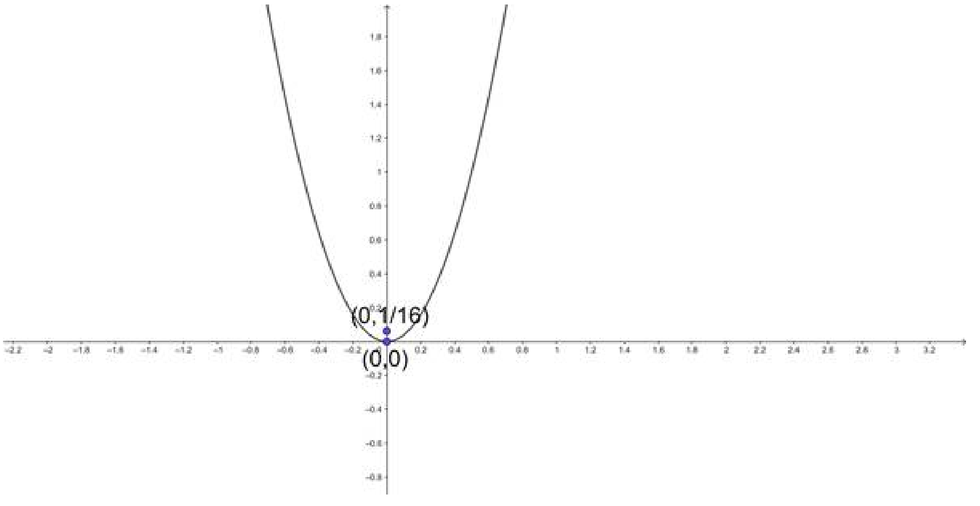
4 B. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$4x^2 = y$

Answer

Given parabola is $4x^2 = y$.



Converting to standard form we get,

$\Rightarrow x^2 = \frac{1}{4}y$

Comparing with standard parabola $x^2 = 4ay$ we get,

$\Rightarrow 4a = \frac{1}{4}$

$\Rightarrow a = \frac{1}{16}$

⇒ The vertex is (0, 0)

⇒ The focus is (0, a) = $\left(0, \frac{1}{16}\right)$

⇒ The equation of the axis is x = 0

⇒ The equation of the directrix is y = - a i.e, $y = \frac{-1}{16}$.

⇒ The length of the latus rectum is 4a = $\frac{1}{4}$.

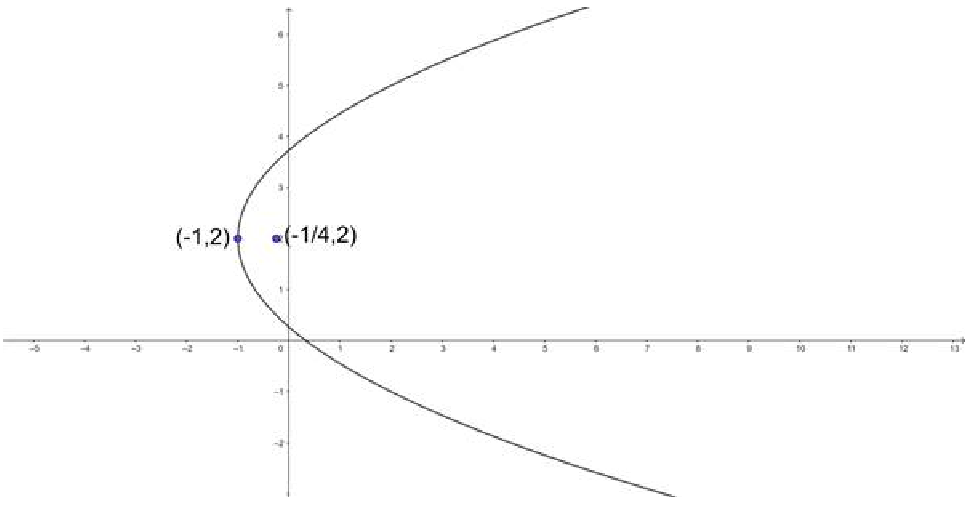
4 C. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$y^2 - 4y - 3x + 1 = 0$

Answer

Given equation of the parabola is $y^2 - 4y - 3x + 1 = 0$



⇒ $y^2 - 4y = 3x - 1$

⇒ $y^2 - 4y + 4 = 3x + 3$

⇒ $(y - 2)^2 = 3(x + 1)$

Comparing with the standard form of parabola $(y - a)^2 = 4b(x - c)$ we get,

⇒ $4b = 3$

⇒ $b = \frac{3}{4}$

⇒ The vertex is $(c, a) = (-1, 2)$

⇒ The focus is $(b + c, a) = \left(\frac{3}{4} - 1, 2\right) = \left(\frac{-1}{4}, 2\right)$

⇒ The equation of the axis is $y - a = 0$ i.e, $y - 2 = 0$

⇒ The equation of the directrix is $x - c = -b$

⇒ Directrix is $x - (-1) = \frac{-3}{4}$

⇒ Directrix is $x = -1 - \frac{3}{4}$

⇒ Directrix is $x = \frac{-7}{4}$

⇒ Length of latus rectum is $4b = 3$.

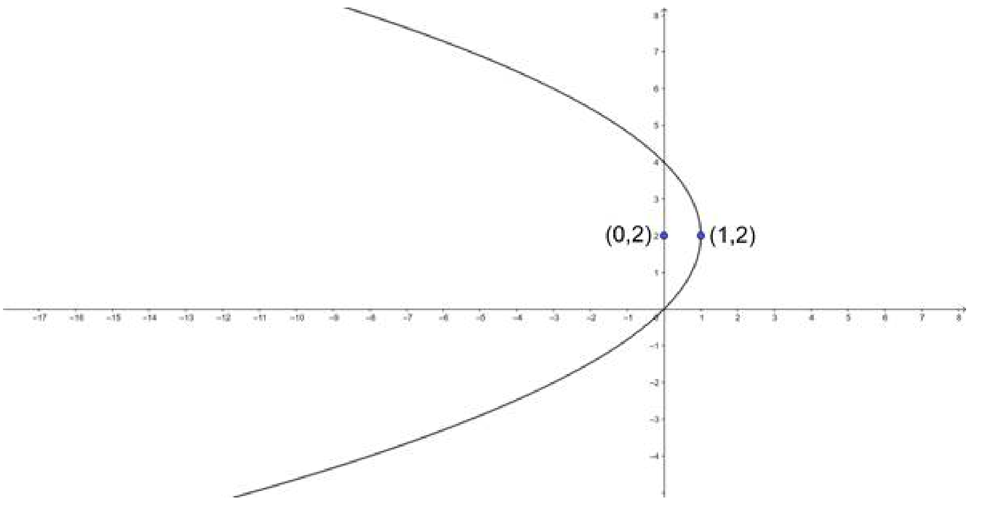
4 D. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$y^2 - 4y + 4x = 0$

Answer

Given equation of the parabola is $y^2 - 4y + 4x = 0$



$\Rightarrow y^2 - 4y = -4x$

$\Rightarrow y^2 - 4y + 4 = -4x + 4$

$\Rightarrow (y - 2)^2 = -4(x - 1)$

Comparing with the standard form of parabola $(y - a)^2 = -4b(x - c)$ we get,

$\Rightarrow 4b = 4$

$\Rightarrow b = 1$

⇒ The vertex is $(c, a) = (1, 2)$

⇒ The focus is $(b + c, a) = (1-1, 2) = (0, 2)$

⇒ The equation of the axis is $y - a = 0$ i.e, $y - 2 = 0$

⇒ The equation of the directrix is $x - c = b$

⇒ Directrix is $x - 1 = 1$

⇒ Directrix is $x = 1 + 1$

⇒ Directrix is $x = 2$

⇒ Length of latus rectum is $4b = 4$.

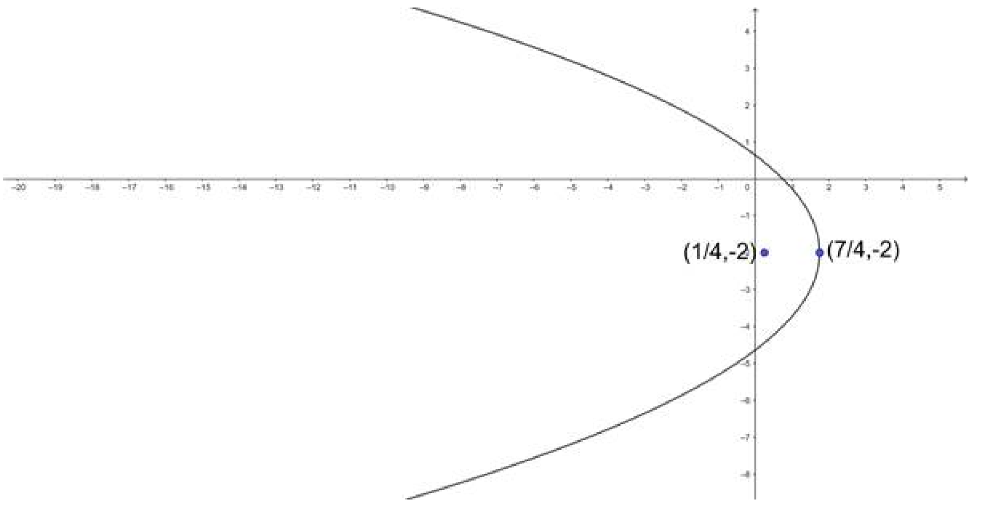
4 E. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$y^2 + 4x + 4y - 3 = 0$

Answer

Given equation of the parabola is $y^2 + 4x + 4y - 3 = 0$



⇒ $y^2 + 4y = -4x + 3$

⇒ $y^2 + 4y + 4 = -4x + 7$

⇒ $(y + 2)^2 = -4\left(x - \frac{7}{4}\right)$

Comparing with the standard form of parabola $(y - a)^2 = -4b(x - c)$ we get,

⇒ $4b = 4$

⇒ $b = 1$

⇒ The vertex is (c, a) = $\left(\frac{7}{4}, -2\right)$

⇒ The focus is (- b + c, a) = $\left(-1 + \frac{7}{4}, -2\right) = \left(\frac{3}{4}, -2\right)$

⇒ The equation of the axis is y - a = 0 i.e, y + 2 = 0

⇒ The equation of the directrix is x - c = b

⇒ Directrix is $x - \frac{7}{4} = 1$

⇒ Directrix is $x = 1 + \frac{7}{4}$

⇒ Directrix is $x = \frac{11}{4}$

⇒ Length of latus rectum is 4b = 4.

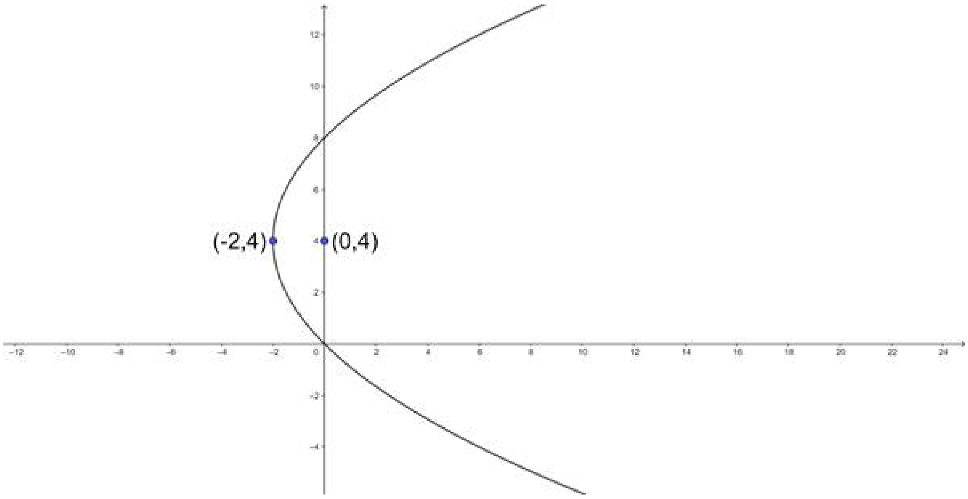
4 F. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$y^2 = 8x + 8y$

Answer

Given equation of the parabola is $y^2 = 8x + 8y$



⇒ $y^2 - 8y = 8x$

⇒ $y^2 - 8y + 16 = 8x + 16$

⇒ $(y - 4)^2 = 8(x + 2)$

Comparing with the standard form of parabola $(y - a)^2 = 4b(x - c)$ we get,

⇒ 4b = 8

⇒ b = 2

⇒ The vertex is (c, a) = (- 2, 4)

⇒ The focus is (b + c, a) = $(2-2,2) = (0,4)$

⇒ The equation of the axis is y - a = 0 i.e, y - 4 = 0

⇒ The equation of the directrix is x - c = - b

⇒ Directrix is x - (-2) = -2

⇒ Directrix is x = -2 -2

⇒ Directrix is x = -4

⇒ Length of latus rectum is 4b = 8.

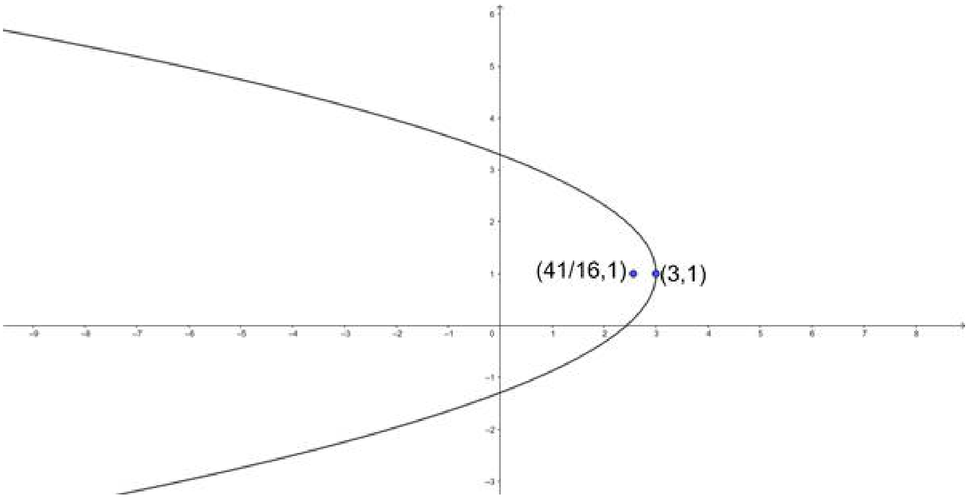
4 G. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$4(y - 1)^2 = - 7(x - 3)$

Answer

Given equation of the parabola is $4(y - 1)^2 = - 7(x - 3)$



$\Rightarrow (y-1)^2 = -\frac{7}{4}(x-3)$

Comparing with the standard form of parabola $(y - a)^2 = - 4b(x - c)$ we get,

$\Rightarrow 4b = \frac{7}{4}$

$\Rightarrow b = \frac{7}{16}$

⇒ The vertex is (c, a) = (3, 1)

⇒ The focus is (- b + c, a) = $\left(\frac{-7}{16} + 3, 1\right) = \left(\frac{41}{16}, 1\right)$

⇒ The equation of the axis is y - a = 0 i.e, y - 1 = 0

⇒ The equation of the directrix is x - c = b

⇒ Directrix is $x - 3 = \frac{7}{16}$

⇒ Directrix is $x = 3 + \frac{7}{16}$

⇒ Directrix is $x = \frac{55}{16}$

⇒ Length of latus rectum is $4b = \frac{7}{4}$.

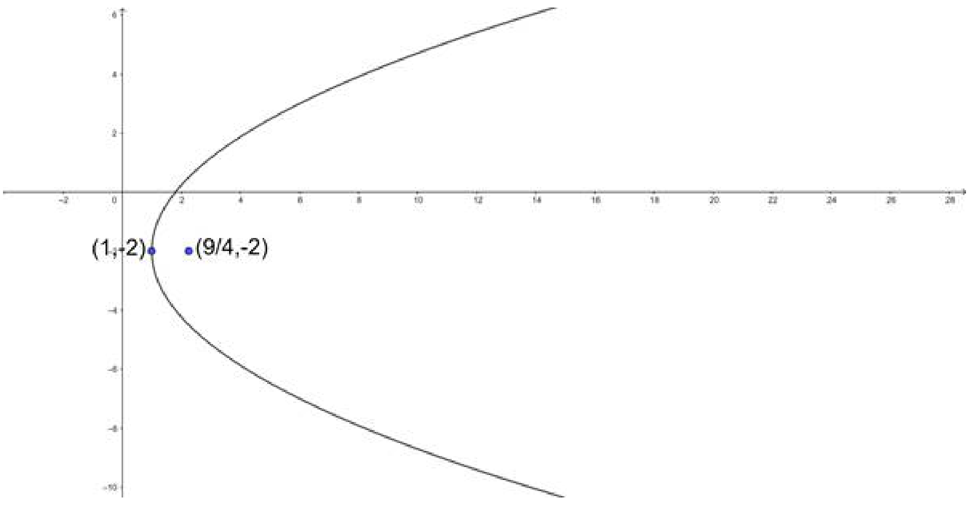
4 H. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$y^2 = 5x - 4y – 9$

Answer

Given equation of the parabola is $y^2 = 5x - 4y - 9$



⇒ $y^2 + 4y = 5x - 9$

⇒ $y^2 + 4y + 4 = 5x - 5$

⇒ $(y + 2)^2 = 5(x - 1)$

Comparing with the standard form of parabola $(y - a)^2 = 4b(x - c)$ we get,

⇒ $4b = 5$

⇒ $b = \frac{5}{4}$

⇒ The vertex is (c, a) = (1, - 2)

⇒ The focus is (b + c, a) = $\left(\frac{5}{4} + 1, -2\right) = \left(\frac{9}{4}, -2\right)$

⇒ The equation of the axis is y - a = 0 i.e, y + 2 = 0

⇒ The equation of the directrix is x - c = - b

⇒ Directrix is $x - 1 = \frac{-5}{4}$

⇒ Directrix is $x = 1 - \frac{5}{4}$

⇒ Directrix is $x = \frac{-1}{4}$

⇒ Length of latus rectum is 4b = 5.

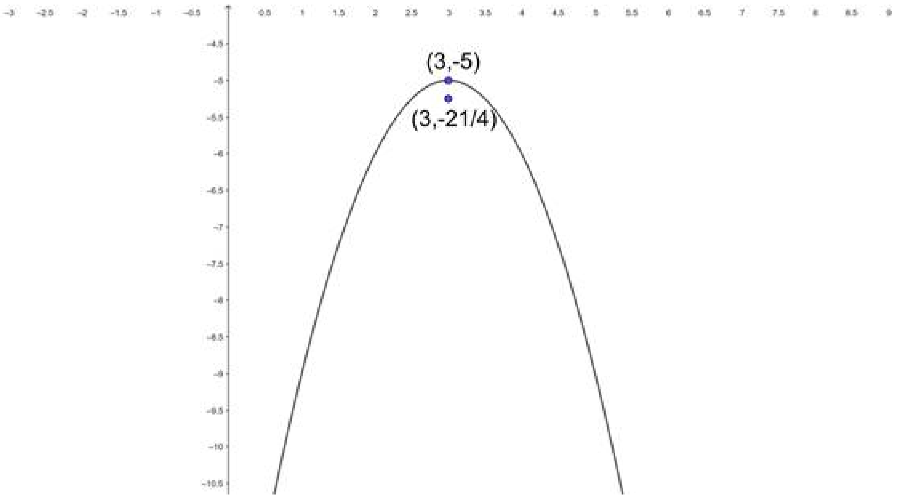
4 I. Question

Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas

$x^2 + y = 6x – 14$

Answer

Given equation of the parabola is $x^2 + y = 6x - 14$



⇒ x^2 - 6x = - y - 14

⇒ x^2 - 6y + 9 = - y - 5

⇒ $(x - 3)^2$ = - (y + 5)

Comparing with the standard form of parabola $(x - a)^2$ = - 4b(y - c) we get,

⇒ 4b = 1

⇒ $b = \frac{1}{4}$

⇒ The vertex is (a, c) = (3, - 5)

⇒ The focus is (a, - b + c) = $\left(3, -\frac{1}{4} - 5\right) = \left(3, -\frac{21}{4}\right)$

⇒ The equation of the axis is x - a = 0 i.e, x - 3 = 0

⇒ The equation of the directrix is y - c = b

⇒ Directrix is $y + 5 = \frac{1}{4}$

⇒ The directrix is $y = -5 + \frac{1}{4}$

⇒ Directrix is $y = \frac{-19}{4}$

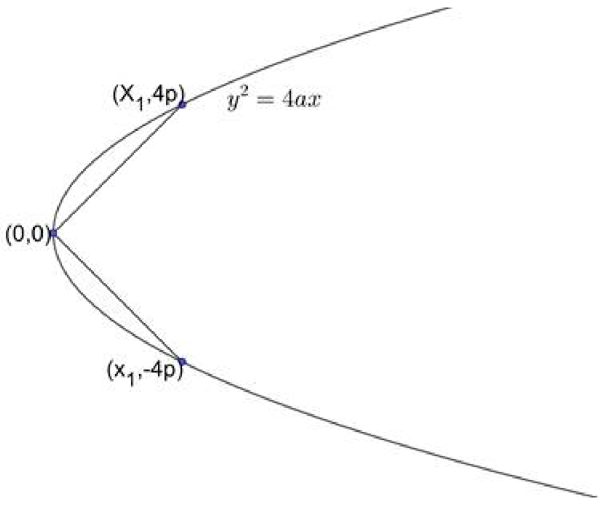
⇒ Length of latus rectum is 4b = 1.

5. Question

For the parabola, y^2 = 4px find the extremities of a double ordinate of length 8p. Prove that the lines from the vertex to its extremities are at right angles.

Answer

Let AB be the double ordinate of length 8p for the parabola y^2 = 4px.



Comparing with standard form we get $y^2 = 4ax$ we get,

⇒ axis is y = 0

⇒ vertex is O(0, 0).

We know that double ordinate is perpendicular to the axis.

Let us assume that the point at which the double ordinate meets the axis is $(x_1, 0)$.

Then the equation of the double ordinate is $y = x_1$. It meets the parabola at the points $(x_1, 4p)$ and $(x_1, -4p)$ as its length is 8p.

Let us find the value of x_1 by substituting in the parabola.

⇒ $(4p)^2 = 4p(x_1)$

⇒ $x_1 = 4p$.

The extremities of the double ordinate are A(4p, 4p) and B(4p, - 4p).

Let us find assume the slopes of OA and OB be m_1 and m_2. Let us find their values.

⇒ $m_1 = \frac{4p-0}{4p-0}$

⇒ $m_1 = \frac{4p}{4p}$

⇒ $m_1 = 1$

⇒ $m_2 = \frac{4p-0}{-4p-0}$

⇒ $m_2 = \frac{4p}{-4p}$

⇒ $m_2 = -1$

⇒ $m_1.m_2 = 1. -1$

⇒ $m_1.m_2 = -1$

We have got product of slopes - 1. So, the lines OA and OB are perpendicular to each other.

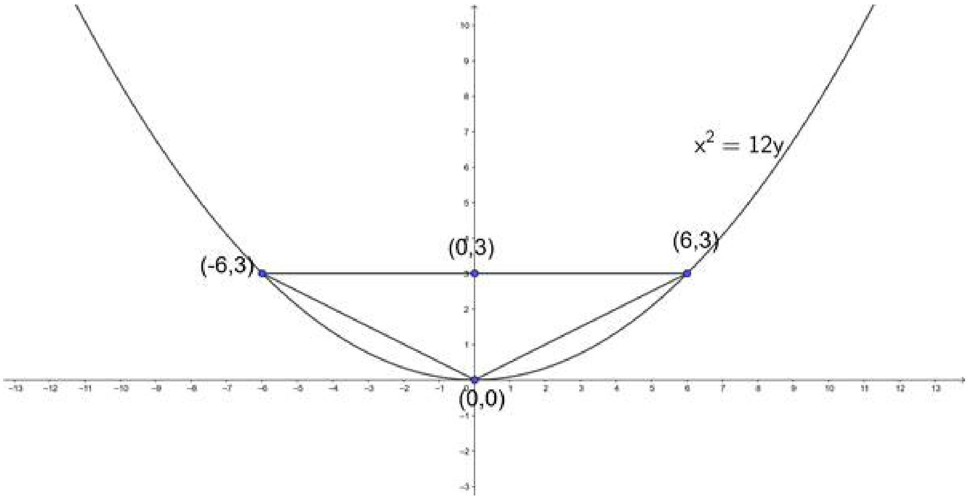
So, the extremities of double ordinate make right angle with vertex.

6. Question

Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus - rectum.

Answer

Given that we need to find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus - rectum.



Comparing it with the standard form of parabola $x^2 = 4by$.

⇒ Vertex is 0(0, 0)

⇒ Ends of latus rectum is (2b, b), (- 2b, b)

⇒ 4b = 12

⇒ b = 3

⇒ Ends of latus rectum is (2(3), 3), (- 2(3), 3)

⇒ Ends of latus rectum is A(6, 3), B(- 6, 3)

We know that area of the triangle with the vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$

$\Rightarrow A = \frac{1}{2}\begin{vmatrix} 0-6 & 0-(-6) \\ 0-3 & 0-3 \end{vmatrix}$

$\Rightarrow A = \frac{1}{2}\begin{vmatrix} -6 & 6 \\ -3 & -3 \end{vmatrix}$

$\Rightarrow A = \frac{1}{2}|(-3 \times -6) - (-3 \times 6)|$

$\Rightarrow A = \frac{1}{2}| + 18 + 18|$

$\Rightarrow A = \frac{1}{2}|36|$

⇒ A = 18sq.units.

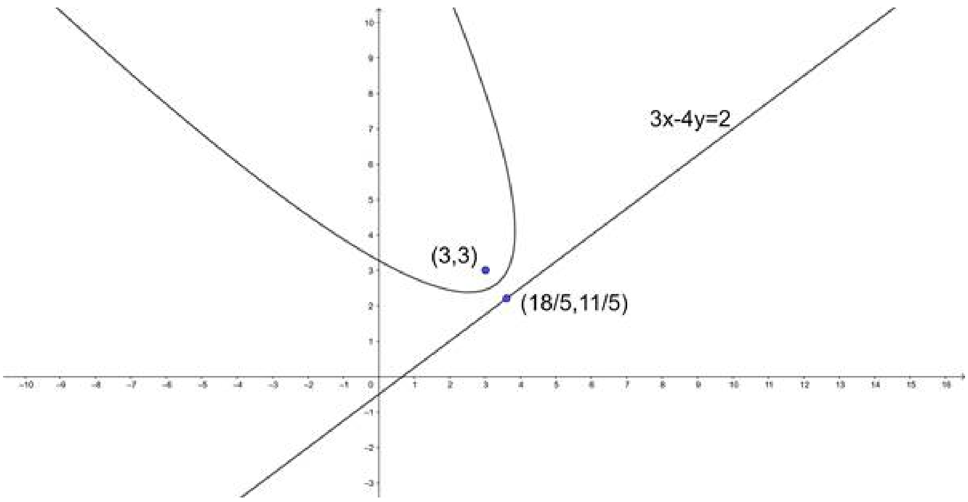
∴The area of the triangle is 18 sq.units.

7. Question

Find the coordinates of the point of intersection of the axis and the directrix of the parabola whose focus is (3, 3) and directrix is 3x – 4y = 2. Find also the length of the latus - rectum.

Answer

Given the equation of directrix is 3x - 4y = 2 and focus is (3, 3).



We know that the directrix and axis are perpendicular to each other. The axis also passes through the focus.

Let us find the slope of the directrix.

We know that the slope of the line ax + by + c = 0 is $\frac{-a}{b}$.

$\Rightarrow m_1 = \frac{-3}{-4}$

$\Rightarrow m_1 = \frac{3}{4}$.

We know that the products of the slopes of the perpendicular lines (non - vertical) is - 1. Let us assume the slope of axis is m_2.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow \frac{3}{4}.m_2 = -1$

$\Rightarrow m_2 = \frac{-4}{3}$.

We know that the equation of the line passing through the point (x_1, y_1) and having slope m is $(y - y_1) = m(x - x_1)$

$\Rightarrow y - 3 = \frac{-4}{3}(x - 3)$

$\Rightarrow 3(y - 3) = - 4(x - 3)$

$\Rightarrow 3y - 9 = - 4x + 12$

$\Rightarrow 4x + 3y = 21$

On solving the lines 4x + 3y = 21 and 3x - 4y = 2, we get the intersection point to be $\left(\frac{18}{5}, \frac{11}{5}\right)$.

We know that the length of latus rectum is equal to the twice of the perpendicular distance between directrix and focus.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow L = 2\frac{|3(3) - 4(3) - 2|}{\sqrt{3^2 + (-4)^2}}$

$\Rightarrow L = 2\frac{|9 - 12 - 2|}{\sqrt{9 + 16}}$

$\Rightarrow L = \frac{2|-5|}{\sqrt{25}}$

$\Rightarrow L = \frac{2 \times 5}{5}$

⇒ L = 2

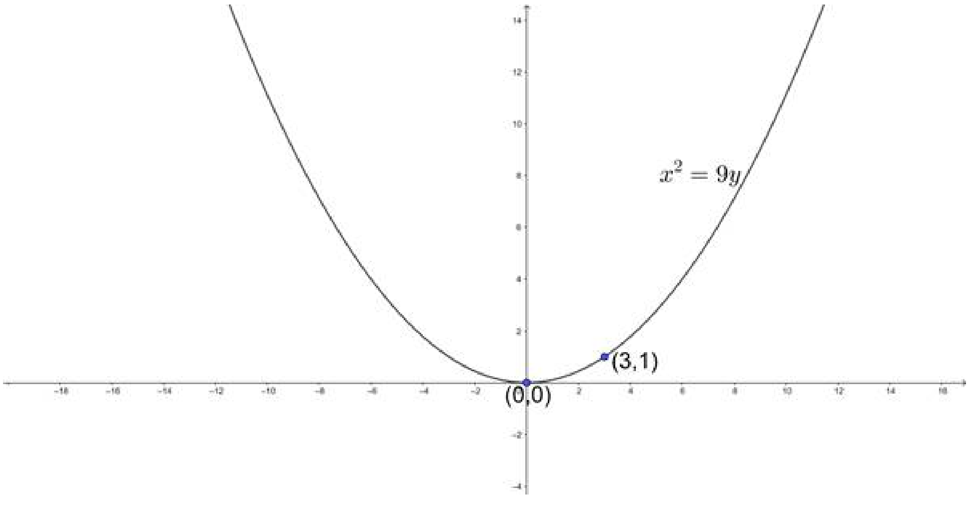
∴The length of the latus rectum is 2.

8. Question

At what point of the parabola $x^2 = 9y$ is the abscissa three times that of ordinate?

Answer

Given that we need to find the point on parabola $x^2 = 9y$ such that the abscissa is three times the ordinate.



Let us assume the point be $(3y_1, y_1)$. Substituting in the parabola we get,

$\Rightarrow (3y_1)^2 = 9(y_1)$

$\Rightarrow 9y_1^2 = 9y_1$

$\Rightarrow y_1^2 - y_1 = 0$

$\Rightarrow y_1(y_1 - 1) = 0$

$\Rightarrow y_1 = 0$ or $y_1 - 1 = 0$

$\Rightarrow y_1 = 0$ or $y_1 = 1$

The points is B(3(1), 1) i.e, (3, 1).

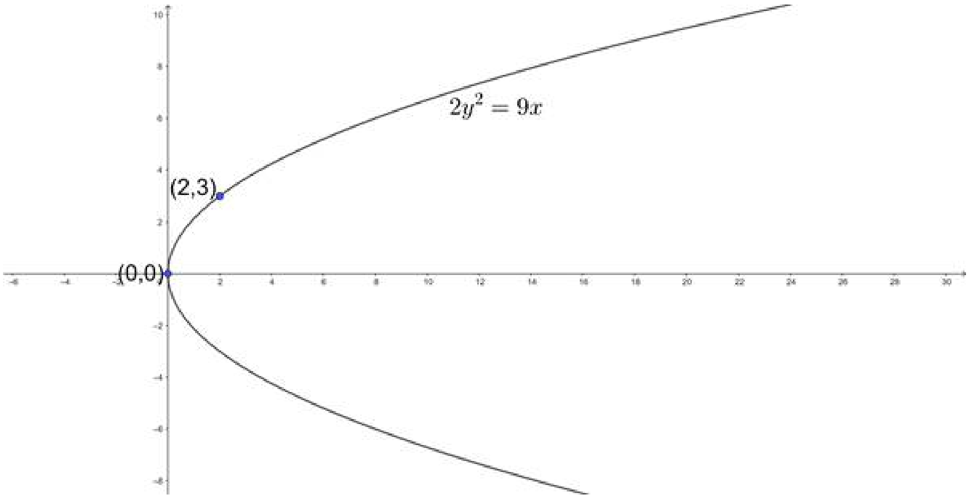
∴The point is (3, 1).

9. Question

Find the equation of a parabola with vertex at the origin, the axis along the x - axis and passing through (2, 3).

Answer

Given that we need to find the equation of the parabola whose vertex is at origin and axis along the x - axis and also passing through (2, 3).



We know that the standard equation of the parabola whose axis is x - axis and vertex at origin is $y^2 = 4ax$.

Substituting (2, 3) in the equation of parabola.

$\Rightarrow (3)^2 = 4a(2)$

$\Rightarrow 9 = 8a$

$\Rightarrow a = \frac{9}{8}$

Now,

$\Rightarrow y^2 = 4\left(\frac{9}{8}\right)x$

$\Rightarrow y^2 = \frac{9}{2}x$

$\Rightarrow 2y^2 = 9x$.

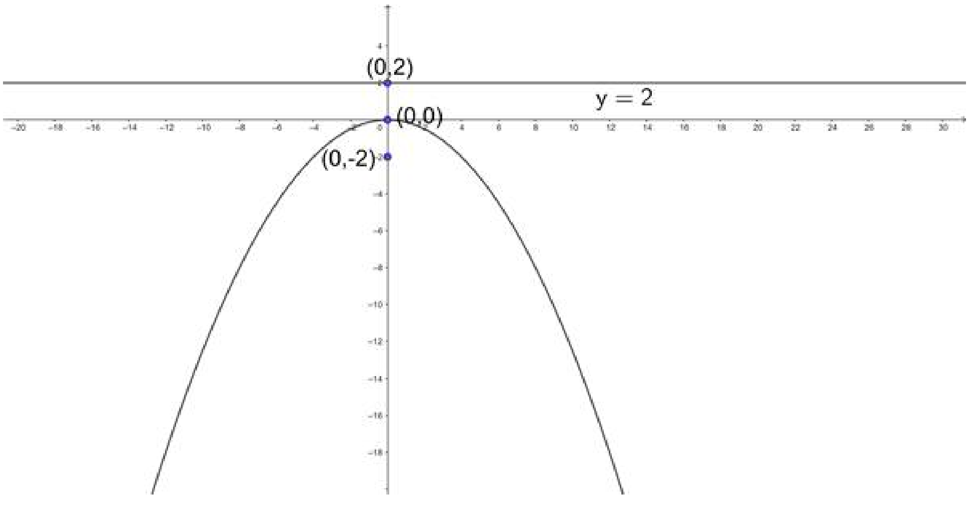
∴The equation of the parabola is $2y^2 = 9x$.

10. Question

Find the equation of a parabola with vertex at the origin and the directrix, y = 2.

Answer

Given that we need to find the equation of the parabola with vertex at the origin and the directrix(M) is y = 2.



We know that the axis is perpendicular to the directrix. Since the directrix is parallel to the x - axis, the axis will parallel to the y - axis.

Since, axis passes through the origin, the equation of the axis is x = 0.

The intersection of the point of the axis and directrix will be (0, 2).

We know that vertex is the midpoint of focus and point on directrix which lies on axis(intersection point).

Let (x_1, y_1) be the focus,

$\Rightarrow (0,0) = \left(\frac{x+0}{2}, \frac{y+2}{2}\right)$

$\Rightarrow \frac{x}{2} = 0 \text{ and } \frac{y+2}{2} = 0$

⇒ x = 0 and y + 2 = 0

⇒ x = 0 and y = - 2.

The focus is S(0, - 2).

Let P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

$\Rightarrow (x-0)^2 + (y-(-2))^2 = \left(\frac{|y-2|}{\sqrt{1^2}}\right)^2$

$\Rightarrow x^2 + y^2 + 4y + 4 = \frac{(|y-2|)^2}{1}$

⇒ $x^2 + y^2 + 4y + 4 = y^2 - 4y + 4$

⇒ $x^2 = - 8y$

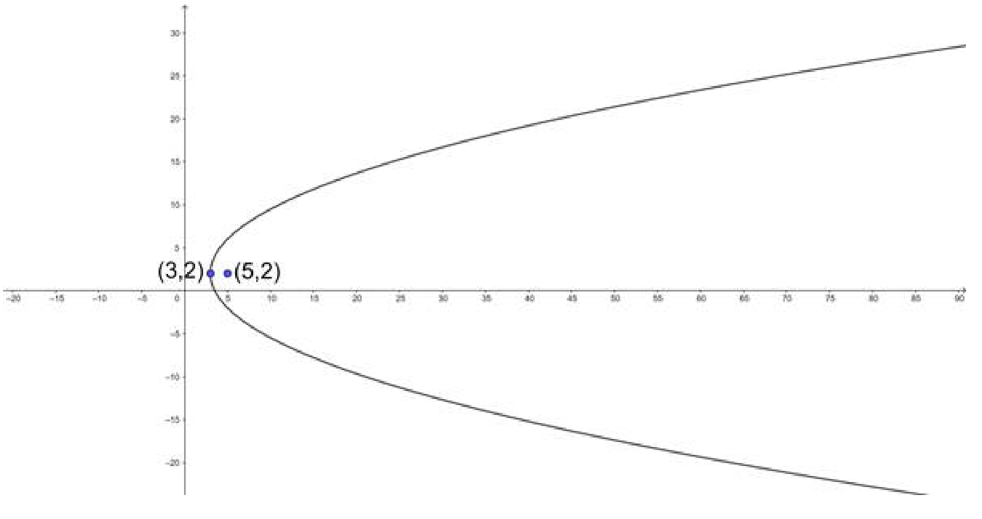
∴The equation of the parabola is $x^2 = - 8y$.

11. Question

Find the equation of the parabola whose focus is (5, 2) and having a vertex at (3, 2).

Answer

Given that we need to find the equation of the parabola whose focus is (5, 2) and having a vertex at (3, 2).



We know that the directrix is perpendicular to the axis and vertex is the midpoint of focus and the intersection point of axis and directrix.

Let us find the slope of the axis. We know that the slope of the straight line passing through the points $(x_1,$

y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

$\Rightarrow m_1 = \frac{2-2}{5-3}$

$\Rightarrow m_1 = \frac{0}{2}$

$\Rightarrow m_1 = 0$.

Here the axis is parallel to the x-axis, so the directrix should be parallel to the y-axis.

Let us assume the intersection point on directrix is (x_1, y_1).

$\Rightarrow (3, 2) = \left(\frac{x+5}{2}, \frac{y+2}{2}\right)$

$\Rightarrow \frac{x+5}{2} = 3 \text{ and } \frac{y+2}{2} = 2$

$\Rightarrow x + 5 = 6$ and $y + 2 = 4$

$\Rightarrow x = 1$ and $y = 2$.

The point on directrix is (1, 2).

We know that the equation parallel to y-axis is x = k. So, the equation of the directrix is x = 1.

Let P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow SP = PM$

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x - 5)^2 + (y - 2)^2 = \left(\frac{|x-1|}{\sqrt{1^2}}\right)^2$

$\Rightarrow x^2 - 10x + 25 + y^2 - 4y + 4 = \frac{(|x-1|)^2}{1}$

$\Rightarrow x^2 - 10x + y^2 - 4y + 29 = x^2 - 2x + 1$

$\Rightarrow y^2 - 8x - 4y + 28 = 0$

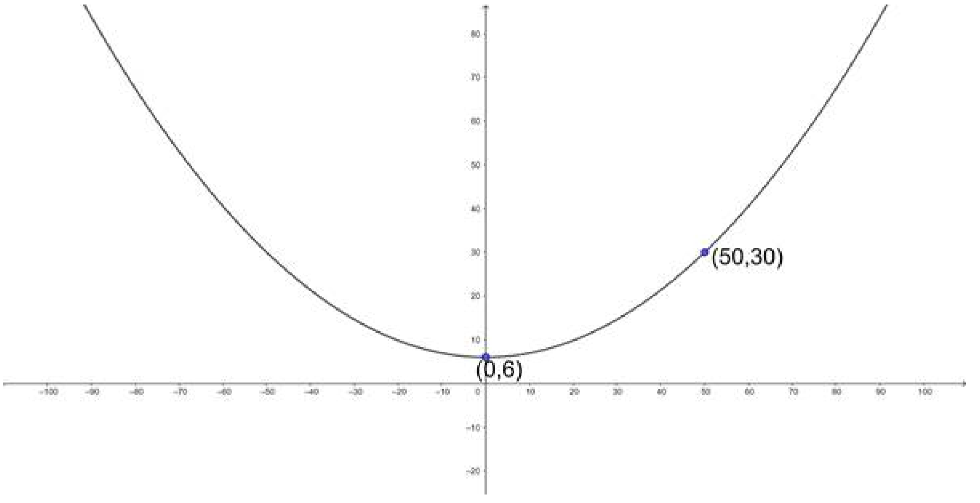
$\therefore$The equation of the parabola is $y^2 - 8x - 4y + 28 = 0$.

12. Question

The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30m and the shortest wire being 6m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Answer

Given that the cable hangs in the form of a parabola.



It is told that the length of the shortest wire supported is 6.

It is clear that the vertex of the parabola is S(0, 6).

We know that the equation of the parabola having (0, a) as vertex is $x^2 = 4b(y - a)$

Let us assume the equation of the parabola is $x^2 = 4b(y - 6)$.

It is told that the road way is 100m long. We usually give maximum support at the middle of the road i.e. at 50m. At the 50m of the road way, the length of the support wire used is 30m.

It is clear that the point (50, 30) lies on parabola, substituting this point in the equation of the parabola, we get,

$\Rightarrow (50)^2 = 4b(30 - 6)$

$\Rightarrow 2500 = 4b(24)$

$\Rightarrow 96b = 2500 \, b = \frac{625}{24}$

The equation of the parabola is $x^2 = \frac{625}{6}(y - 6)$.

We need to find the length of the support that is needed to give at the 18m in the roadway.

Let us assume the length of the support is l m,

We have a point (18, l) on the parabola, substituting in the equation we get,

$\Rightarrow 18^2 = \frac{625}{6}(l - 6)$

$\Rightarrow \frac{324 \times 6}{625} = l - 6$

$\Rightarrow 3.11 = l - 6$

$\Rightarrow l = 9.11m$

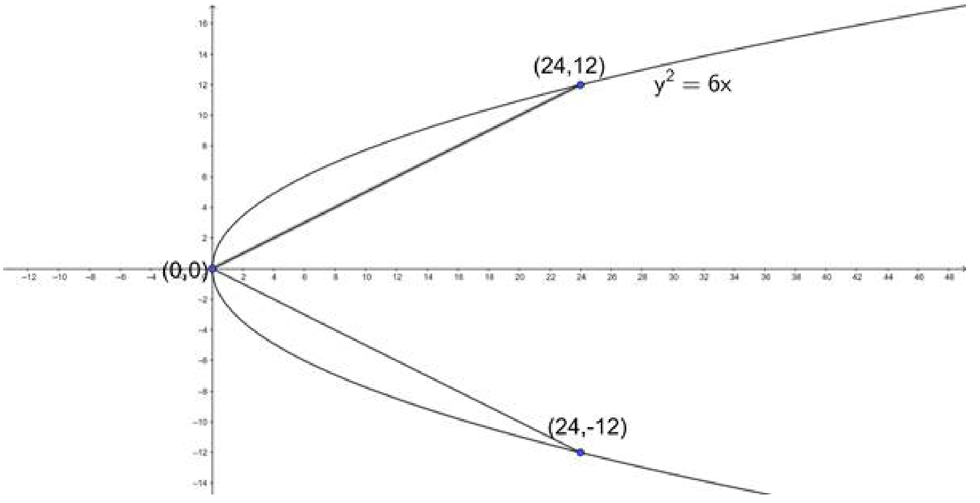
∴The length of the support required is 9.11m.

13. Question

Find the equations of the lines joining the vertex of the parabola $y^2 = 6x$ to the point on it which have abscissa 24.

Answer

Given that we need to find the equations of the lines joining the vertex to the point whose abscissa is 24 on the parabola $y^2 = 6x$.



We know that for a parabola $y^2 = 4ax$, the vertex is (0, 0).

So, the vertex of the parabola $y^2 = 6x$ is (0, 0).

Let us assume the point on parabola be (24, l).

Substituting in the parabola we get,

$\Rightarrow l^2 = 6(24)$

$\Rightarrow l^2 = 144$

$\Rightarrow l = \sqrt{144}$

$\Rightarrow l = \pm 12.$

The points on the parabola is (24, ±12).

Let us find the equation of the line passing though the points (0, 0) and (24, 12).

We know that the equation of the straight lines passing through the points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - 0 = \frac{12-0}{24-0}(x - 0)$

$\Rightarrow y = \frac{12}{24}x$

$\Rightarrow x = 2y.$

∴The equation of the line is $x = 2y$.

Let us find the equation of the line passing though the points (0, 0) and (24, - 12).

We know that the equation of the straight lines passing through the points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - 0 = \frac{-12-0}{24-0}(x - 0)$

$\Rightarrow y = \frac{-12}{24}x$

$\Rightarrow x = -2y.$

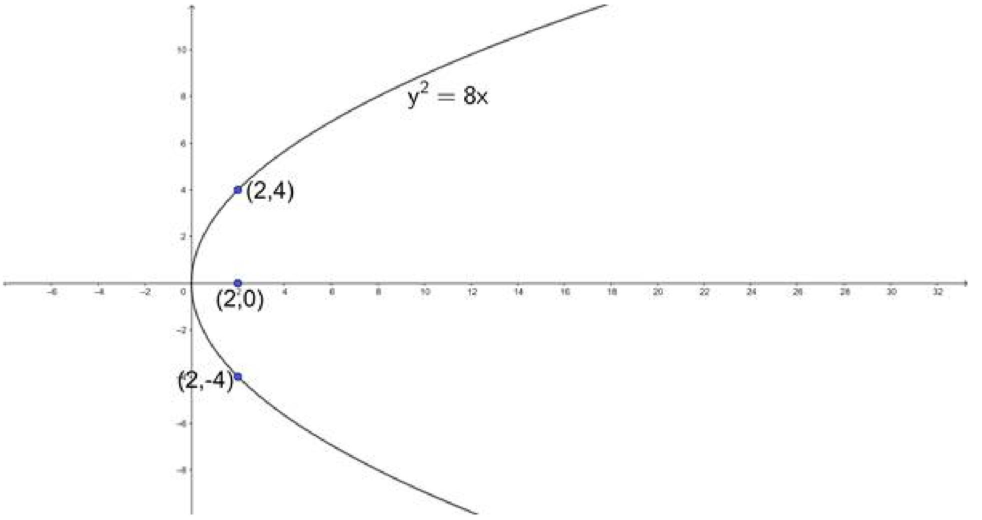
∴The equation of the line is $x = -2y$.

14. Question

Find the coordinates of points on the parabola $y^2 = 8x$ whose focal distance is 4.

Answer

Given that we need to find the coordinates of points on the parabola $y^2 = 8x$ whose focal distance is 4.



We know that the focal distance is the distance from the focus to any point on the parabola.

Comparing the given parabola with standard parabola $y^2 = 4ax$. We get,

$\Rightarrow 4a = 8$

$\Rightarrow a = 2$

$\Rightarrow$ focus = (a, 0) = (2, 0)

We know that point on $y^2 = 4ax$ is represented by $(at^2, 2at)$, where t is any real number.

The point on $y^2 = 8x$ is $(2t^2, 4t)$

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow 4 = \sqrt{(2 - 2t^2)^2 + (0 - 4t)^2}$

$\Rightarrow 16 = 4 + 4t^4 - 8t^2 + 16t^2$

$\Rightarrow 16 = 4t^4 + 8t^2 + 4$

$\Rightarrow 4 = t^4 + 2t^2 + 1$

$\Rightarrow 4 = (t^2 + 1)^2$

$\Rightarrow t^2 + 1 = 2$

$\Rightarrow t^2 = 1$

$\Rightarrow t = \pm 1$

The points on parabola is,

$\Rightarrow (2t^2, 4t) = (2(\pm 1)^2, 4(\pm 1))$

$\Rightarrow (2t^2, 4t) = (2, \pm 4)$

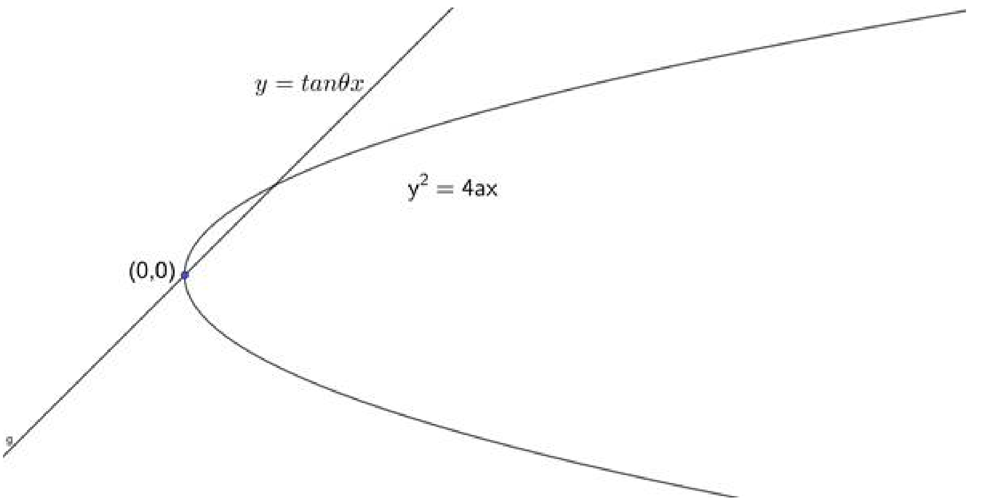
∴The points on parabola is $(2, \pm 4)$.

15. Question

Find the length of the line segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where the line - segment makes an angle θ to the x - axis.

Answer

Given that we need to find the length of the line joining the vertex of parabola $y^2 = 4ax$ and a point on the parabola where the line segment makes an angle θ to the x - axis.



We know that vertex of the parabola is (0, 0).

We know that the equation of the line passing through origin and making angle θ to the x - axis is given by y = (tanθ)x.

Substituting y value in the equation of parabola we get,

⇒ $(x\tan\theta)^2 = 4ax$

⇒ $x^2\tan^2\theta = 4ax$

⇒ $x\tan^2\theta = 4a$

⇒ $x = \frac{4a}{\tan^2\theta}$

⇒ $y = \tan\theta x$

⇒ $y = \tan\theta\left(\frac{4a}{\tan^2\theta}\right)$

⇒ $y = \frac{4a}{\tan\theta}$

The point on the parabola is $\left(\frac{4a}{\tan^2\theta}, \frac{4a}{\tan\theta}\right)$.

We know that the distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

⇒ $S = \sqrt{\left(0 - \frac{4a}{\tan^2\theta}\right)^2 + \left(0 - \frac{4a}{\tan\theta}\right)^2}$

⇒ $S = \sqrt{\frac{16a^2}{\tan^4\theta} + \frac{16a^2}{\tan^2\theta}}$

⇒ $S = \sqrt{\frac{16a^2}{\tan^4\theta}(1 + \tan^2\theta)}$

⇒ $S = \frac{4a^2}{\tan^2\theta}\sqrt{1 + \tan^2\theta}$

⇒ $S = \frac{4a^2\cos\theta}{\tan\theta\sin\theta}\sqrt{\sec^2\theta}$

⇒ $S = \frac{4a^2\cos\theta\sec\theta}{\tan\theta\sin\theta}$

⇒ $S = \frac{4a^2}{\tan\theta\sin\theta}$

⇒ $S = 4a^2\cot\theta\text{cosec}\theta$.

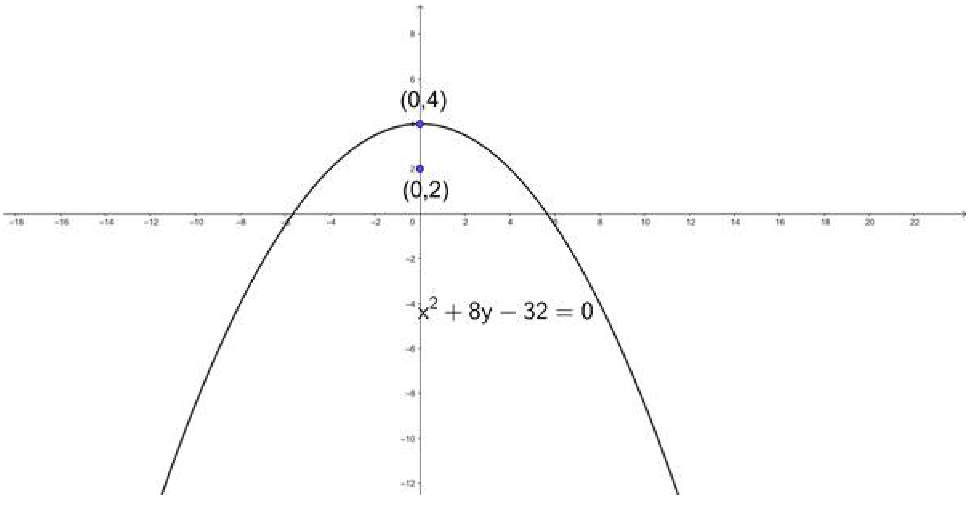
∴The distance is $4a^2\cot\theta\text{cosec}\theta$.

16. Question

If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola.

Answer

Given that we need to find the equation of the parabola whose focus is (0, 2) and having a vertex at (0, 4).



We know that the directrix is perpendicular to the axis and vertex is the midpoint of focus and the intersection point of axis and directrix.

Let us find the slope of the axis. We know that the slope of the straight line passing through the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$.

$\Rightarrow m_1 = \frac{4-2}{0-0}$

$\Rightarrow m_1 = \frac{2}{0}$

$\Rightarrow m_1 = \infty$.

Here the axis is parallel to the y - axis, so the directrix should be parallel to the x - axis.

Let us assume the intersection point on directrix is (x_1, y_1).

$\Rightarrow (0,4) = \left(\frac{x+0}{2}, \frac{y+2}{2}\right)$

$\Rightarrow \frac{x}{2} = 0 \text{ and } \frac{y+2}{2} = 4$

$\Rightarrow x = 0$ and $y + 2 = 8$

$\Rightarrow x = 0$ and $y = 6$.

The point on directrix is (0, 6).

We know that equation parallel to x - axis is y = k. So, the equation of the directrix is y = 6.

Let P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow SP = PM$

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x-0)^2 + (y-2)^2 = \left(\frac{|y-6|}{\sqrt{1^2}}\right)^2$

$\Rightarrow x^2 + y^2 - 4y + 4 = \frac{(|y-6|)^2}{1}$

$\Rightarrow x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$

$\Rightarrow x^2 + 8y - 32 = 0$

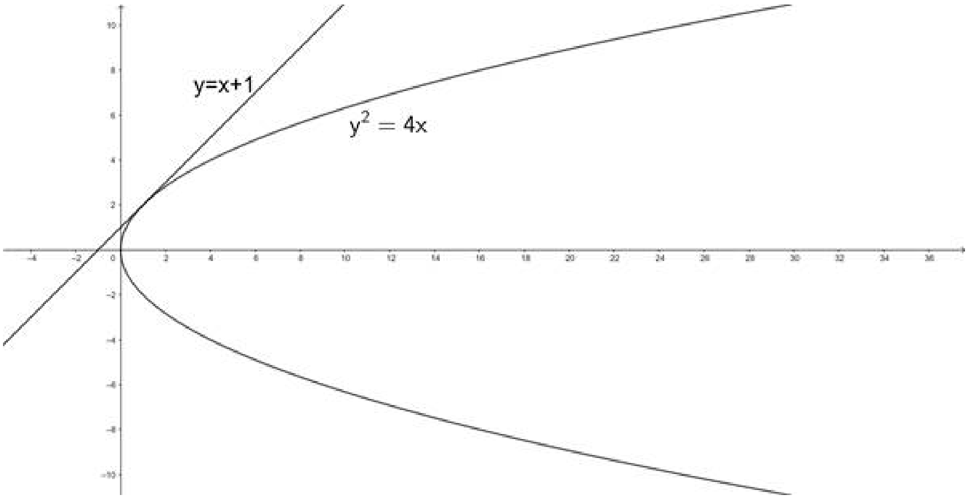
∴The equation of the parabola is $x^2 + 8y - 32 = 0$.

17. Question

If the line y = mx + 1 is tangent to the parabola $y^2 = 4x$, then find the value of m.

Answer

Given that the line y = mx + 1 is the tangent to the parabola $y^2 = 4x$. We need to find the value of m.



Let us substitute the value in the equation of parabola.

$\Rightarrow (mx + 1)^2 = 4x$

$\Rightarrow m^2x^2 + 2mx + 1 = 4x$

$\Rightarrow m^2x^2 + (2m - 4)x + 1 = 0$

The quadratic will have similar roots if the line is tangent to the parabola.

We know that for a quadratic equation $ax^2 + bx + c = 0$ to have equal roots, the condition to be satisfied is $b^2 - 4ac = 0$

$\Rightarrow (2m - 4)^2 - 4(m^2)(1) = 0$

$\Rightarrow 4m^2 - 16m + 16 - 4m^2 = 0$

$\Rightarrow 16 - 16m = 0$

$\Rightarrow 16m = 16$

$\Rightarrow m = 1$

∴The value of m is 1.

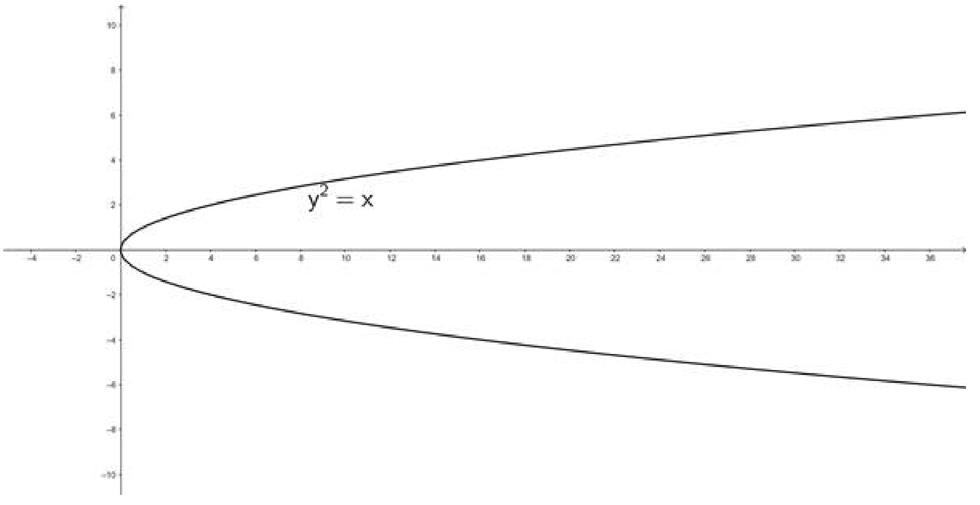
Very Short Answer

1. Question

Write the axis of symmetry of the parabola $y^2 = x$.

Answer

Given equation of the parabola is $y^2 = x$.



Comparing with the standard form of parabola $y^2 = 4ax$,

⇒ The axis of parabola is $y = 0$.

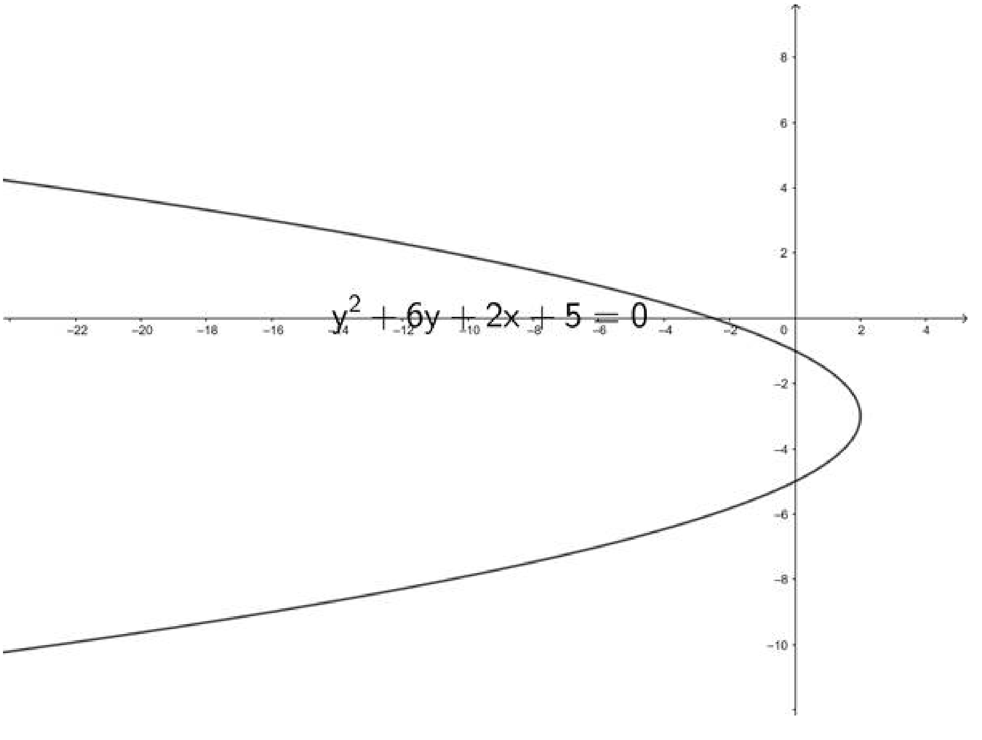
∴The axis of parabola is $y = 0$.

2. Question

write the distance between the vertex and focus of the parabola $y^2 + 6y + 2x + 5 = 0$

Answer

Given equation of the parabola is $y^2 + 6y + 2x + 5 = 0$



$\Rightarrow y^2 + 6y + 5 = -2x$

$\Rightarrow y^2 + 6y + 9 = -2x + 4$

$\Rightarrow (y + 3)^2 = -2(x - 2)$

Comparing with standard form of parabola $(y - a)^2 = -4b(x - c)$ we get,

$\Rightarrow 4b = 2$

$\Rightarrow b = \frac{2}{4}$

$\Rightarrow b = \frac{1}{2}$

We know that the distance between the vertex and focus is b.

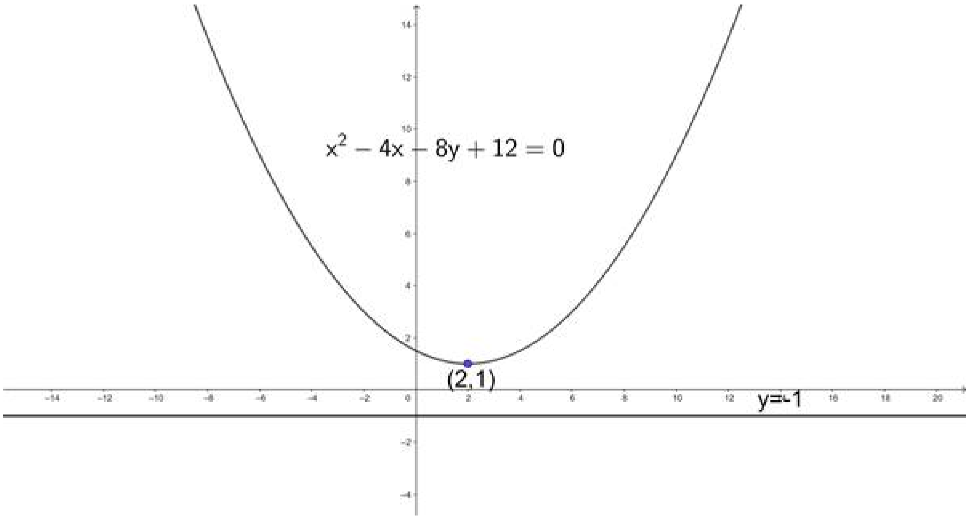
∴The distance between the vertex and focus is $\frac{1}{2}$.

3. Question

Write the equation of the directrix of the parabola $x^2 - 4x - 8y + 12 = 0$

Answer

Given equation of the parabola is $x^2 - 4x - 8y + 12 = 0$



$\Rightarrow x^2 - 4x + 12 = 8y$

$\Rightarrow x^2 - 4x + 4 = 8y - 8$

$\Rightarrow (x - 2)^2 = 8(y - 1)$

Comparing with the standard form of parabola $(x - a)^2 = 4b(y - c)$ we get,

$\Rightarrow 4b = 8$

$\Rightarrow b = 2$

⇒ The equation of the directrix is $y - c = - b$

⇒ Directrix is $y - 1 = - 2$

⇒ Directrix is $y = - 2 + 1$

⇒ Directrix is $y = - 1$

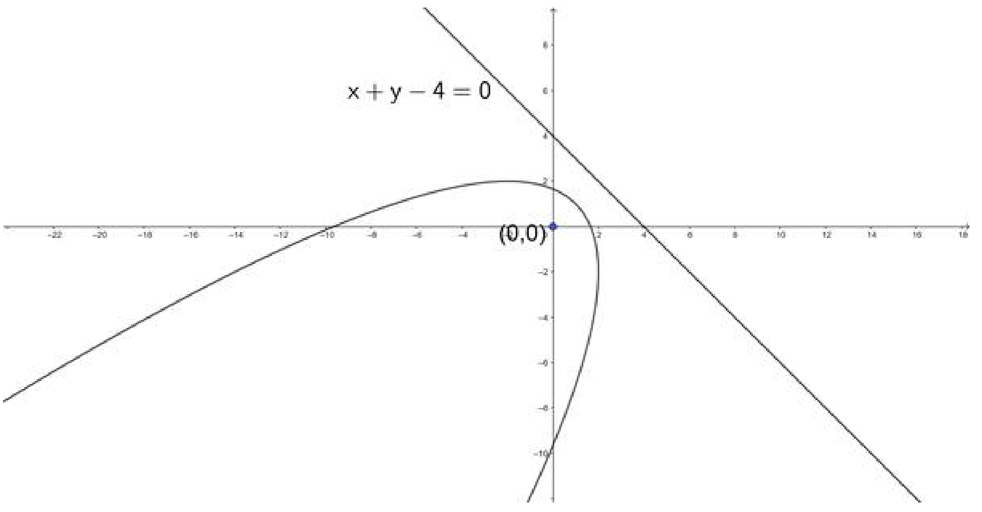
∴The equation of the directrix is $y = - 1$.

4. Question

Write the equation of the parabola with focus (0, 0) and directrix $x + y - 4 = 0$.

Answer

Given that we need to find the equation of the parabola whose focus is S(0, 0) and directrix(M) is $x + y - 4 = 0$.



Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 0)^2 + (y - 0)^2 = \left(\frac{|x + y - 4|}{\sqrt{1^2 + 1^2}}\right)^2$

⇒ $x^2 + y^2 = \frac{(|x + y - 4|)^2}{1 + 1}$

⇒ $x^2 + y^2 = \frac{(x^2 + y^2 + 16 - 8x - 8y + 2xy)}{2}$

⇒ $2x^2 + 2y^2 = x^2 + y^2 - 8x - 8y + 2xy + 16$

⇒ $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$

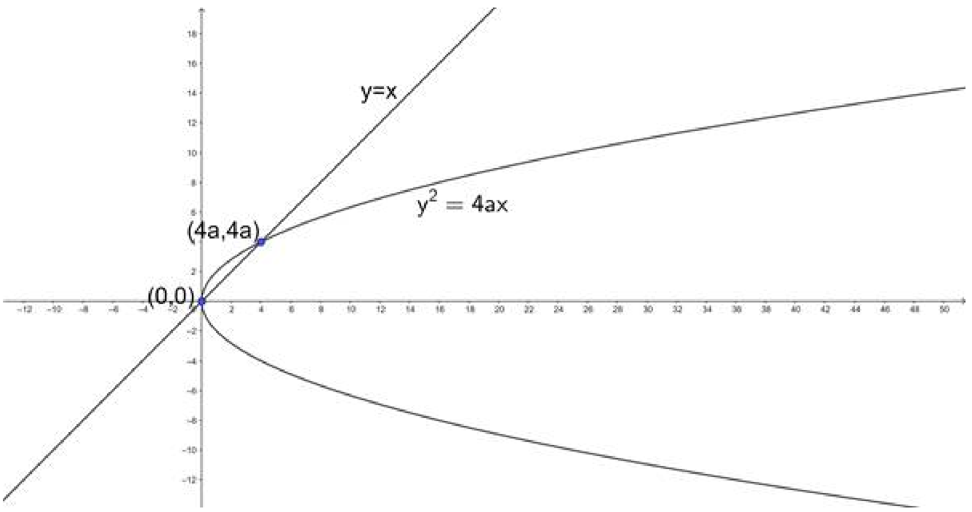
∴The equation of the parabola is $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$.

5. Question

Write the length of the chord of the parabola $y^2 = 4ax$ which passes through the vertex and is inclined to the axis at π/4.

Answer

Given that we need to find the length of the chord of the parabola $y^2 = 4ax$ which passes through the vertex and is inclined to axis at $\frac{\pi}{4}$. The figure for the parabola is as follows:



We know that the vertex and axis of the parabola $y^2 = 4ax$ is (0, 0) and y = 0(x - axis) respectively.

We know that the equation of the straight line passing through the origin and inclines to the x - axis at an angle θ is y = tanθx.

$\Rightarrow y = \tan\left(\frac{\pi}{4}\right)x$

⇒ y = 1.x

⇒ y = x.

The equation of the chord is y = x.

Substituting y = x in the equation of parabola.

$\Rightarrow x^2 = 4ax$

⇒ x = 4a.

⇒ y = x = 4a

The chord passes through the points (0, 0) and (4a, 4a).

We know that the distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow l = \sqrt{(0 - 4a)^2 + (0 - 4a)^2}$

$\Rightarrow l = \sqrt{16a^2 + 16a^2}$

$\Rightarrow l = \sqrt{32a^2}$

$\Rightarrow l = 4\sqrt{2}a$

∴The length of the chord is 4√2a units.

6. Question

If b and c are lengths of the segments of any focal chord of the parabola $y^2 = 4ax$, then write the length of its latus - rectum.

Answer

Given that b and c are lengths of the segments of any focal chord of the parabola $y^2 = 4ax$.

We know that the length of the latus - rectum is 4a.

We know that the semi - length of the latus - rectum is the harmonic mean of any length of any focal chord.

We know that if a, b, c are in harmonic progression, harmonic mean is given by,

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{b}$

Here the length of semi-latus rectum is 2a.

$\Rightarrow \frac{2}{2a} = \frac{1}{b} + \frac{1}{c}$

$\Rightarrow \frac{1}{a} = \frac{b+c}{bc}$

$\Rightarrow a = \frac{bc}{b+c}$

Length of the latus - rectum is 4a,

$\Rightarrow 4a = \frac{4bc}{b+c}$

∴The length of the latus - rectum is $\frac{4bc}{b+c}$.

7. Question

PSQ is a focal chord of the parabola $y^2 = 8x$. If SP = 6, then write SQ.

Answer

Given that PSQ is a focal chord of the parabola $y^2 = 8x$.

It is also given that SP = 6. We need to find the value of SQ.

Comparing with the standard form of parabola $y^2 = 4ax$

⇒ 4a = 8

⇒ a = 2

We know that the semi - length of the latus - rectum is the harmonic mean of any length of any focal chord.

We know that if a, b, c are in harmonic progression, harmonic mean is given by,

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{b}$

We know that the semi - length of the latus - rectum is 2a = 4.

$\Rightarrow \frac{2}{2a} = \frac{1}{SP} + \frac{1}{SQ}$

$\Rightarrow \frac{1}{2} = \frac{1}{6} + \frac{1}{SQ}$

$\Rightarrow \frac{1}{3} = \frac{1}{SQ}$

⇒ SQ = 3

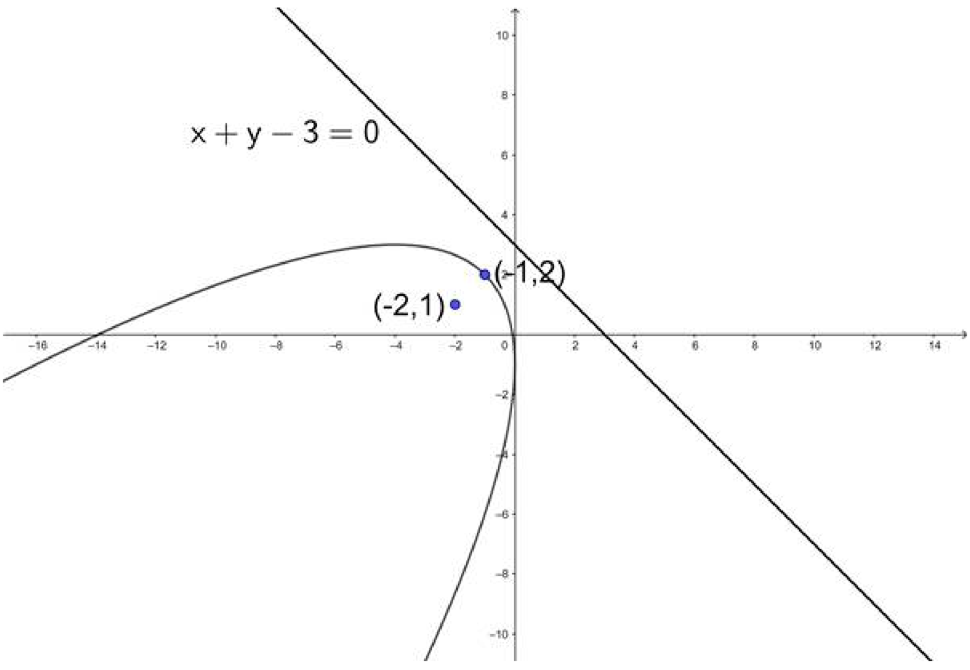
∴The value of SQ is 3.

8. Question

Write the coordinates of the vertex of the parabola whose focus is at (- 2, 1) and directrix is the line x + y – 3 = 0.

Answer

Given that we need to find the equation of the parabola whose focus is S(- 2, 1) and directrix(M) is x + y - 3 = 0.



We know that the directrix is perpendicular to the axis and vertex is the midpoint of focus and the intersection point of axis and directrix.

Let us find the slope of directrix. We know that the slope of the straight line ax + by + c = 0 is $\frac{-a}{b}$.

$\Rightarrow m_1 = \frac{-1}{1}$

$\Rightarrow m_1 = -1$

We know that the product of slopes of the perpendicular lines is - 1.

Let m_2 be the slope of the directrix.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow -1 \times m_2 = -1$

$\Rightarrow m_2 = 1$

We know that the equation of the straight line passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

$\Rightarrow y - 1 = 1(x - (-2))$

$\Rightarrow y - 1 = x + 2$

$\Rightarrow x - y + 3 = 0$

On solving the lines x - y + 3 = 0 and x + y - 3 = 0 we get the intersection point to be (0, 3).

Let us assume the vertex be (x_1, y_1).

$\Rightarrow (x_1, y_1) = \left(\frac{-2+0}{2}, \frac{1+3}{2}\right)$

$\Rightarrow (x_1, y_1) = (-1, 2)$

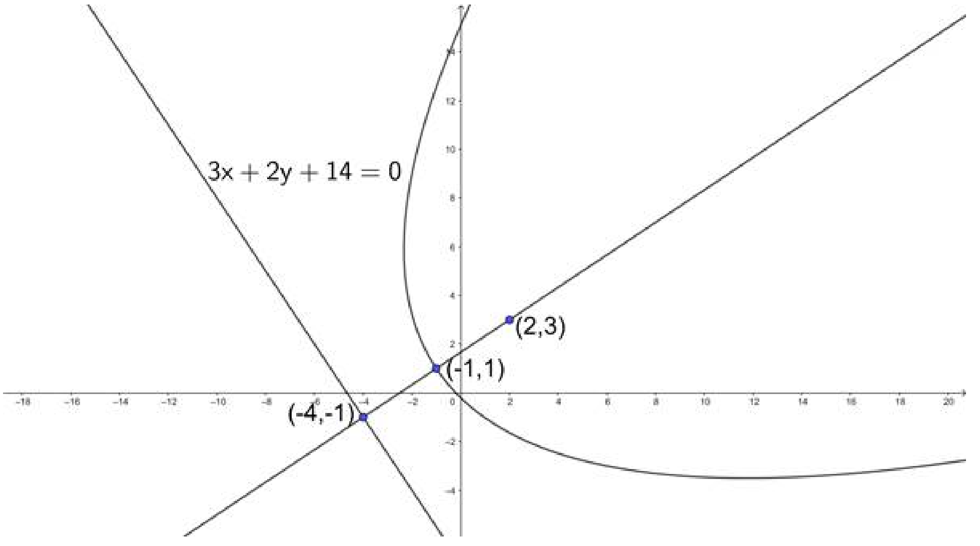
∴The coordinates of the vertex is (- 1, 2).

9. Question

If the coordinates of the vertex and focus of a parabola are (- 1, 1) and (2 , 3) respectively then write the equation of its directrix.

Answer

Given that we need to find the equation of the directrix of a parabola whose focus is (2, 3) and having a vertex at (- 1, 1).



We know that the directrix is perpendicular to the axis and vertex is the midpoint of focus and the intersection point of axis and directrix.

Let us find the slope of the axis. We know that the slope of the straight line passing through the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$.

$\Rightarrow m_1 = \frac{1-3}{-1-2}$

$\Rightarrow m_1 = \frac{-2}{-3}$

$\Rightarrow m_1 = \frac{2}{3}$.

We know that the product of slopes of the perpendicular lines is - 1.

Let m_2 be the slope of the directrix.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow \frac{2}{3} \times m_2 = -1$

$\Rightarrow m_2 = \frac{-3}{2}$

Let us assume the intersection point on directrix is (x_1, y_1).

$\Rightarrow (-1, 1) = \left(\frac{x+2}{2}, \frac{y+3}{2}\right)$

$\Rightarrow \frac{x+2}{2} = -1 \text{ and } \frac{y+3}{2} = 1$

$\Rightarrow x + 2 = -2$ and $y + 3 = 2$

$\Rightarrow x = -4$ and $y = -1$.

The point on directrix is (- 4, - 1).

We know that equation of the straight line passing through point (x_1, y_1) and slope m is $y - y_1 = m(x - x_1)$.

$\Rightarrow y - (-1) = \frac{-3}{2}(x - (-4))$

$\Rightarrow 2(y + 1) = -3(x + 4)$

$\Rightarrow 2y + 2 = -3x - 12$

$\Rightarrow 3x + 2y + 14 = 0$

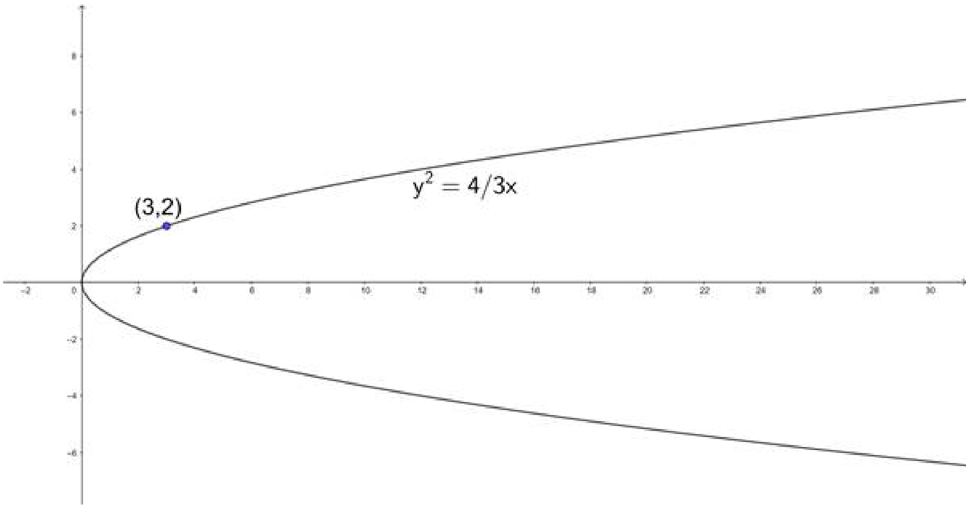
∴The equation of the directrix is 3x + 2y + 14 = 0.

10. Question

If the parabola $y^2 = 4ax$ passes through the point (3, 2), then find the length of its latus - rectum.

Answer

Given that we need to find the length of the latus - rectum of the parabola $y^2 = 4ax$ which passes through the point (3, 2).



We know that the length of the latus - rectum of parabola $y^2 = 4ax$ is 4a.

Substituting the given point in the equation of parabola we get,

$\Rightarrow (2)^2 = 4a(3)$

$\Rightarrow 4 = 4a(3)$

$\Rightarrow 4a = \frac{4}{3}$

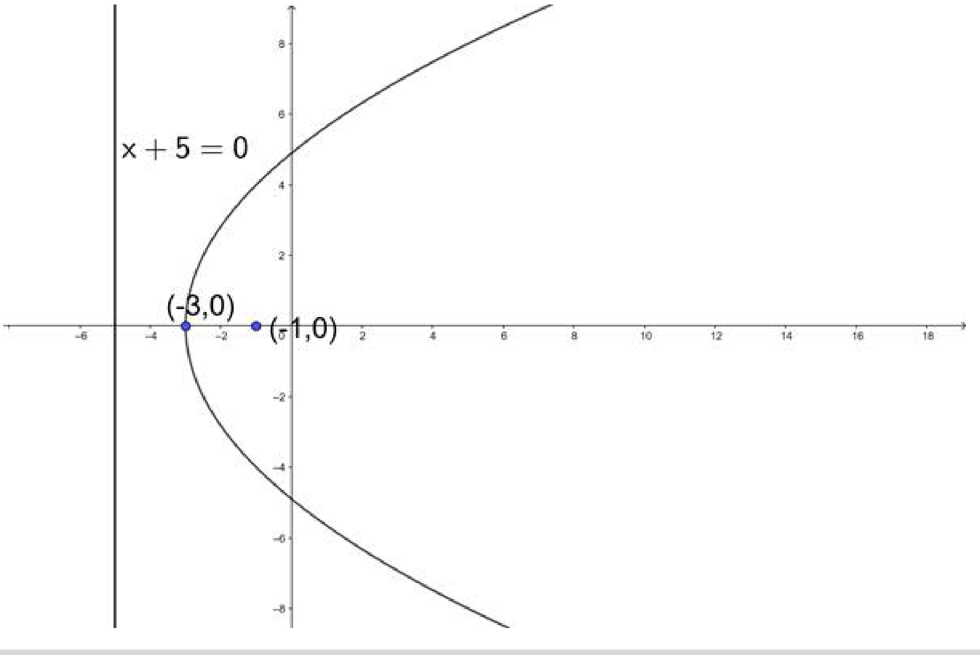
∴The length of the latus - rectum is $\frac{4}{3}$.

11. Question

Write the equation of the parabola whose vertex is at (- 3, 0) and the directrix is x + 5 = 0.

Answer

Given the equation of directrix(M) is x + 5 = 0 and vertex is (- 3, 0).



We know that the directrix and axis are perpendicular to each other. The axis also passes through the vertex.

Let us find the slope of the directrix.

We know that the slope of the line ax + by + c = 0 is $\frac{-a}{b}$.

$\Rightarrow m_1 = \frac{-1}{0}$

$\Rightarrow m_1 = \infty$.

The slope of the directrix is parallel to the slope of the y - axis. So, the slope of the axis is parallel to the slope of x - axis i.e., 0.

We know that the equation of the line passing through the point (x_1, y_1) and having slope m is $(y - y_1) = m(x - x_1)$

$\Rightarrow y - 0 = 0(x - (-3))$

$\Rightarrow y - 0 = 0$

$\Rightarrow y = 0$

On solving the lines x + 5 = 0 and y = 0, we get the intersection point to be (- 5, 0).

We know that vertex is the mid - point of focus and point of intersection of directrix and axis.

Let (x_1, y_1) be the focus.

$\Rightarrow (-3, 0) = \left(\frac{-5 + x_1}{2}, \frac{0 + y_1}{2}\right)$

$\Rightarrow \frac{-5 + x_1}{2} = -3 \text{ and } \frac{y_1}{2} = 0$

$\Rightarrow -5 + x_1 = -6$ and $y_1 = 0$

$\Rightarrow x_1 = -1$ and $y_1 = 0$.

The focus is S(- 1, 0).

Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow SP = PM$

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x - (-1))^2 + (y - 0)^2 = \left(\frac{|x + 5|}{\sqrt{1^2}}\right)^2$

$\Rightarrow x^2 + 2x + 1 + y^2 = \frac{(|x + 5|)^2}{1}$

$\Rightarrow x^2 + y^2 + 2x + 1 = x^2 + 10x + 25$

$\Rightarrow y^2 - 8x - 24 = 0$

$\therefore$The equation of the parabola is $y^2 - 8x - 24 = 0$.

MCQ

1. Question

The coordinates of the focus of the parabola $y^2 – x – 2y + 2 = 0$

A. $\left(\frac{5}{4}, 1\right)$

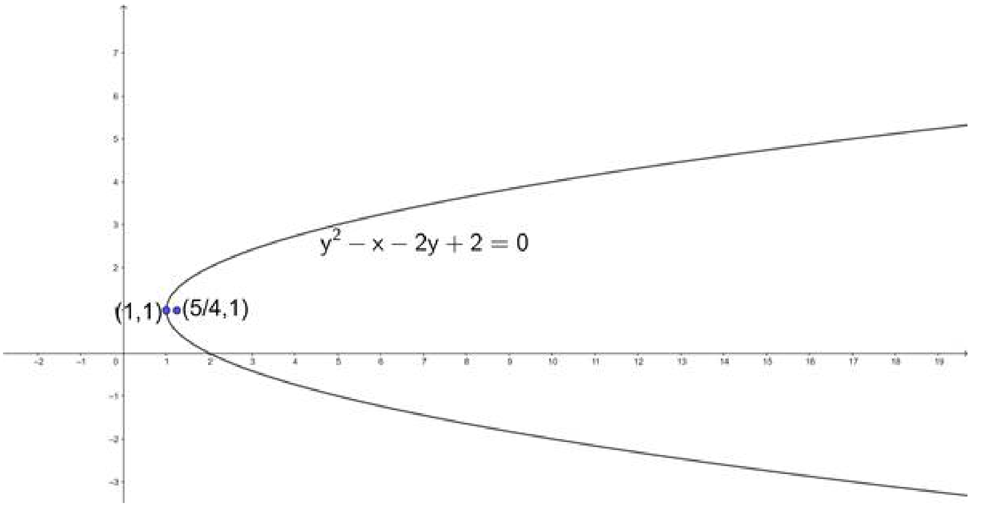
B. $\left(\frac{1}{4}, 0\right)$

C. (1, 1)

D. none of these

Answer

Given equation of the parabola is $y^2 - x - 2y + 2 = 0$



$\Rightarrow y^2 - 2y + 2 = x$

$\Rightarrow y^2 - 2y + 1 = x - 1$

$\Rightarrow (y - 1)^2 = (x - 1)$

Comparing with the standard form of parabola $(y - a)^2 = 4b(x - c)$ we get,

$\Rightarrow 4b = 1$

$\Rightarrow b = \frac{1}{4}$

$\Rightarrow$ The focus is $(b + c, a) = \left(\frac{1}{4} + 1, 1\right) = \left(\frac{5}{4}, 1\right)$

∴The correct option is A

2. Question

The vertex of the parabola $(y + a)^2 = 8a(x – a)$ is

A. (- a, - a)

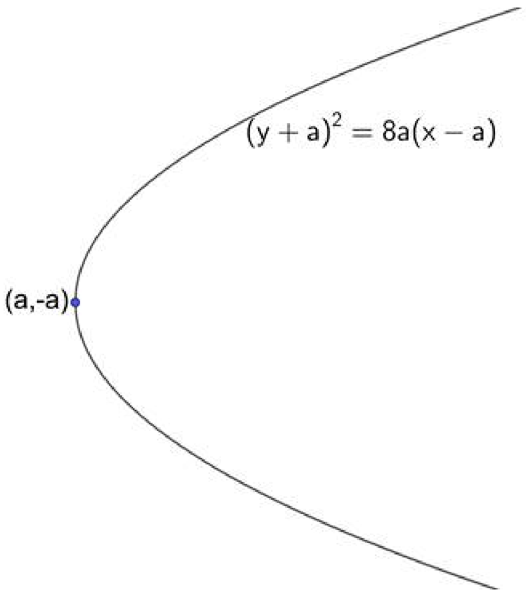
B. (a, - a)

C. (- a, a)

D. none of these

Answer

Given that we need to find the vertex of the parabola $(y + a)^2 = 8a(x - a)$.



Comparing with the standard form of parabola $(y - a)^2 = 4b(x - c)$ we get,

⇒ Vertex of the parabola = (c, a) = (a, - a)

∴The correct option is B

3. Question

If the focus of a parabola is (- 2, 1) and the directrix has the equation x + y = 3, then its vertex is

A. (0, 3)

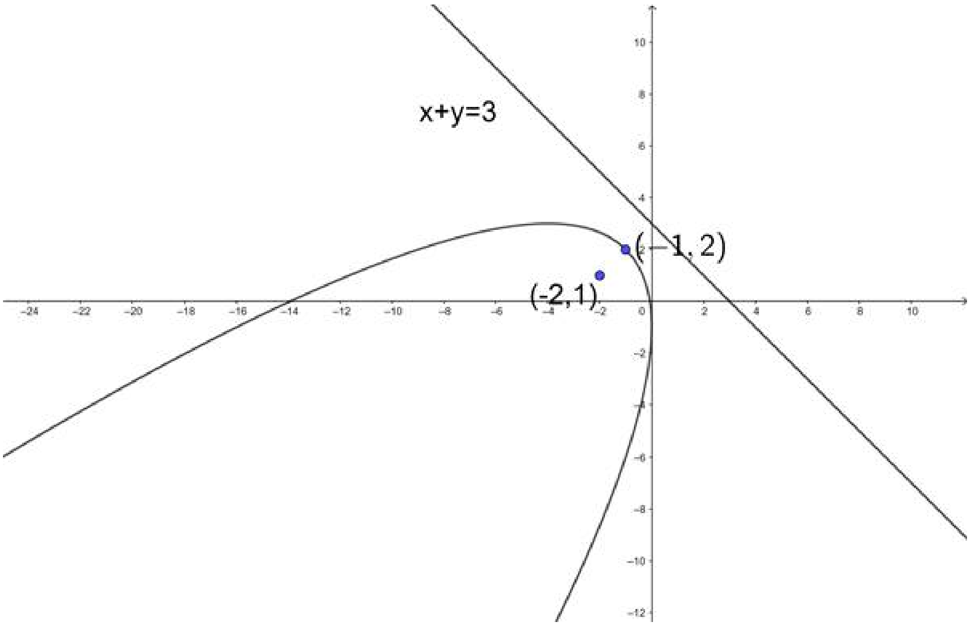
B. $\left(-1, \frac{1}{2}\right)$

C. (- 1, 2)

D. (2, - 1)

Answer

Given the equation of directrix is x + y = 3 and focus is (- 2, 1).



We know that the directrix and axis are perpendicular to each other. The axis also passes through the focus.

Let us find the slope of the directrix.

We know that the slope of the line ax + by + c = 0 is $\frac{-a}{b}$.

$\Rightarrow m_1 = \frac{-1}{1}$

$\Rightarrow m_1 = -1$.

We know that the products of the slopes of the perpendicular lines (non - vertical) is - 1. Let us assume the slope of axis be m_2.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow -1.m_2 = -1$

$\Rightarrow m_2 = 1$.

We know that the equation of the line passing through the point (x_1, y_1) and having slope m is $(y - y_1) = m(x - x_1)$

$\Rightarrow y - 1 = 1(x - (-2))$

$\Rightarrow y - 1 = x + 2$

$\Rightarrow x - y + 3 = 0$

On solving the lines x - y + 3 = 0 and x + y = 3, we get the intersection point to be (0, 3).

We know that vertex is the mid - point of focus and point of intersection of axis and directrix.

Let (x_1, y_1) be the vertex of the parabola.

$\Rightarrow (x_1, y_1) = \left(\frac{0-2}{2}, \frac{3+1}{2}\right)$

$\Rightarrow (x_1, y_1) = \left(\frac{-2}{2}, \frac{4}{2}\right)$

$\Rightarrow (x_1, y_1) = (-1, 2)$

∴The correct option is C

4. Question

The equation of the parabola whose vertex is (a, 0) and the directrix has the equation x + y = 3a, is

A. $x^2 + y^2 + 2xy + 6ax + 10ay + 7a^2 = 0$

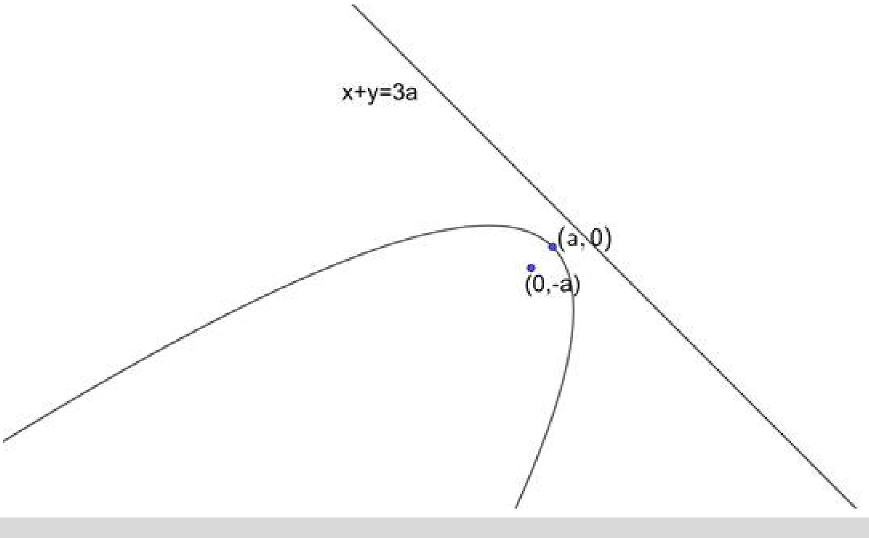
B. $x^2 - 2xy + y^2 + 6ax + 10ay - 7a^2 = 0$

C. $x^2 - 2xy + y^2 - 6ax + 10ay - 7a^2 = 0$

D. none of these

Answer

Given the equation of directrix(M) is x + y = 3a and vertex is (a, 0).



We know that the directrix and axis are perpendicular to each other. The axis also passes through the vertex.

Let us find the slope of the directrix.

We know that the slope of the line ax + by + c = 0 is $\frac{-a}{b}$.

$\Rightarrow m_1 = \frac{-1}{1}$

$\Rightarrow m_1 = -1$.

We know that the product of the slopes of the perpendicular lines is - 1.

Let us assume m_2 be the slope of the axis.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow (-1).m_2 = -1$

$\Rightarrow m_2 = 1$

We know that the equation of the line passing through the point (x_1, y_1) and having slope m is $(y - y_1) = m(x - x_1)$

$\Rightarrow y - 0 = 1(x - a)$

$\Rightarrow y = x - a$

$\Rightarrow x - y - a = 0$

On solving the lines x + y = 3a and x - y - a = 0, we get the intersection point to be (2a, a).

We know that vertex is the mid - point of focus and point of intersection of directrix and axis.

Let (x_1, y_1) be the focus.

$\Rightarrow (a, 0) = \left(\frac{2a + x_1}{2}, \frac{a + y_1}{2}\right)$

$\Rightarrow \frac{2a + x_1}{2} = a$ and $\frac{a + y_1}{2} = 0$

$\Rightarrow 2a + x_1 = 2a$ and $a + y_1 = 0$

$\Rightarrow x_1 = 0$ and $y_1 = -a$.

The focus is S(0, - a).

Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow SP = PM$

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x - 0)^2 + (y - (-a))^2 = \left(\frac{|x + y - 3a|}{\sqrt{1^2 + 1^2}}\right)^2$

$\Rightarrow x^2 + y^2 + 2ay + a^2 = \frac{(|x + y - 3a|)^2}{1 + 1}$

$\Rightarrow 2x^2 + 2y^2 + 4ay + 2a^2 = x^2 + y^2 + 9a^2 + 2xy - 6ax - 6ay$

$\Rightarrow x^2 + y^2 - 2xy + 6ax + 10ay - 7a^2 = 0$

∴The correct option is B

5. Question

The parametric equations of a parabola are $x = t^2 + 1$, $y = 2t + 1$. The Cartesian equation of its directrix is

A. $x = 0$

B. $x + 1 = 0$

C. $y = 0$

D. none of these

Answer

Given the parametric equations of a parabola $x = t^2 + 1$ and $y = 2t + 1$.

Consider $\left(\frac{y-1}{2}\right)^2$,

$\Rightarrow \left(\frac{y-1}{2}\right)^2 = t^2$

$\Rightarrow \left(\frac{y-1}{2}\right)^2 = t^2 + 1 - 1$

$\Rightarrow \left(\frac{y-1}{2}\right)^2 = x - 1$

$\Rightarrow (y - 1)^2 = 4(x - 1)$

Comparing with the standard form of parabola $(y - a)^2 = 4b(x - c)$ we get,

$\Rightarrow 4b = 4$

$\Rightarrow b = 1$

$\Rightarrow$ The equation of the directrix is $x - c = - b$

$\Rightarrow$ Directrix is $x - 1 = - 1$

$\Rightarrow$ Directrix is $x = 1 - 1$

$\Rightarrow$ Directrix is $x = 0$

∴The correct option is A

6. Question

If the coordinates of the vertex and the focus of a parabola are (- 1, 1) and (2, 3) respectively, then the equation of its directrix is

A. $3x + 2y + 14 = 0$

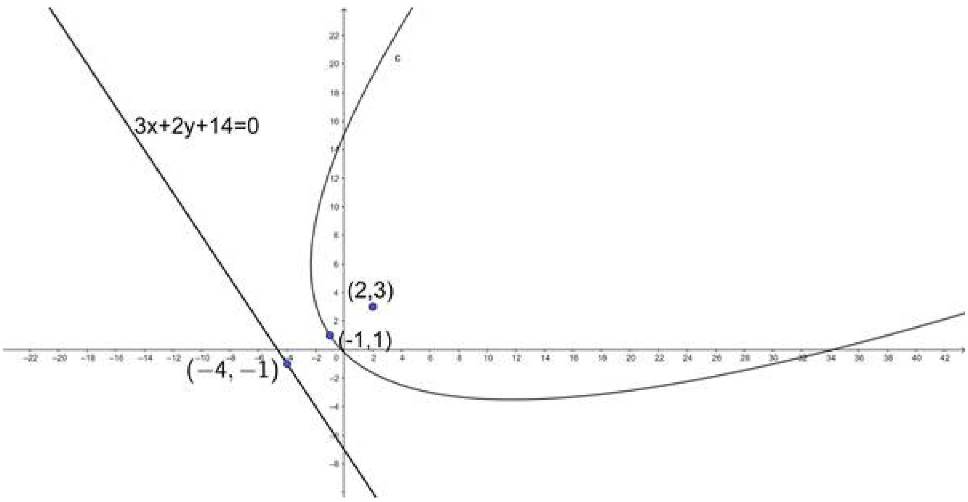
B. $3x + 2y - 25 = 0$

C. $2x - 3y + 10 = 0$

D. None of these

Answer

Given that we need to find the equation of the directrix of a parabola whose focus is (2, 3) and having a vertex at (- 1, 1).



We know that the directrix is perpendicular to the axis and vertex is the midpoint of focus and the intersection point of axis and directrix.

Let us find the slope of the axis. We know that the slope of the straight line passing through the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$.

$\Rightarrow m_1 = \frac{1-3}{-1-2}$

$\Rightarrow m_1 = \frac{-2}{-3}$

$\Rightarrow m_1 = \frac{2}{3}$.

We know that the product of slopes of the perpendicular lines is - 1.

Let m_2 be the slope of the directrix.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow \frac{2}{3} \times m_2 = -1$

$\Rightarrow m_2 = \frac{-3}{2}$

Let us assume the intersection point on directrix is (x_1, y_1).

$\Rightarrow (-1, 1) = \left(\frac{x+2}{2}, \frac{y+3}{2}\right)$

$\Rightarrow \frac{x+2}{2} = -1 \text{ and } \frac{y+3}{2} = 1$

$\Rightarrow x + 2 = -2$ and $y + 3 = 2$

$\Rightarrow x = -4$ and $y = -1$.

The point on directrix is (- 4, - 1).

We know that equation of the straight line passing through point (x_1, y_1) and slope m is $y - y_1 = m(x - x_1)$.

$\Rightarrow y - (-1) = \frac{-3}{2}(x - (-4))$

$\Rightarrow 2(y + 1) = -3(x + 4)$

$\Rightarrow 2y + 2 = -3x - 12$

$\Rightarrow 3x + 2y + 14 = 0$

∴The correct option is A

7. Question

The locus of the points of trisection of the double ordinates of a parabola is a

A. Pair of lines

B. Circle

C. Parabola

D. Straight line

Answer

Let us assume that the parabola be $y^2 = 4ax$.

The parametric equations of the points on the parabola are $(at^2, 2at)$.

If we assume the points of extremities of the double ordinate of the parabola $(at^2, 2at)$ and $(at^2, -2at)$.

We know that the point of trisection is the point in between these point at a ratio of 2:1 or 1:2.

Let us take the ratio to be 1:2.

Let us assume the point of trisection be (x, y).

$$\Rightarrow (x,y) = \left(\frac{(1(at^2)) + (2(at^2))}{1+2}, \frac{(1(-2at)) + (2(2at))}{1+2}\right)$$

$$\Rightarrow (x,y) = \left(\frac{3at^2}{3}, \frac{2at}{3}\right)$$

$$\Rightarrow (x,y) = \left(at^2, \frac{2}{3}at\right)$$

Consider y^2,

$$\Rightarrow y^2 = \left(\frac{2}{3}at\right)^2$$

$$\Rightarrow y^2 = \frac{4}{9}a^2t^2$$

$$\Rightarrow y^2 = \frac{4}{9}a(at^2)$$

$$\Rightarrow y^2 = \frac{4}{9}ax$$

The locus of point of trisection is parabola.

∴The correct option is C

8. Question

The equation of the directrix of the parabola whose vertex and focus are (1, 4) and (2, 6) respectively is

A. x + 2y = 4

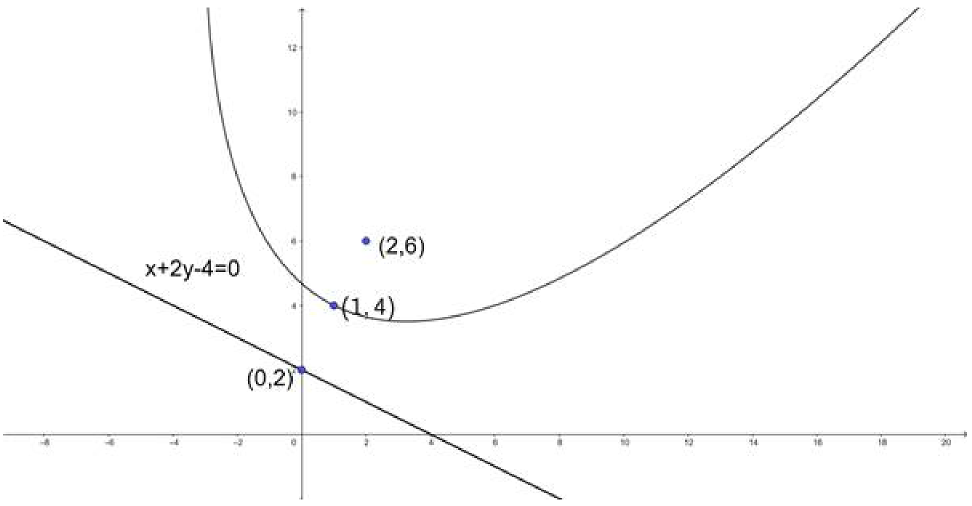
B. x - y = 3

C. 2x + y = 5

D. X + 3y = 8

Answer

Given that we need to find the equation of the directrix of a parabola whose focus is (2, 6) and having a vertex at (1, 4).



We know that the directrix is perpendicular to the axis and vertex is the midpoint of focus and the intersection point of axis and directrix.

Let us find the slope of the axis. We know that the slope of the straight line passing through the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$.

$\Rightarrow m_1 = \frac{6-4}{2-1}$

$\Rightarrow m_1 = \frac{2}{1}$

$\Rightarrow m_1 = 2.$

We know that the product of slopes of the perpendicular lines is - 1.

Let m_2 be the slope of the directrix.

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow 2 \times m_2 = -1$

$\Rightarrow m_2 = \frac{-1}{2}$

Let us assume the intersection point on directrix is (x_1, y_1).

$\Rightarrow (1,4) = \left(\frac{x_1+2}{2}, \frac{y_1+6}{2}\right)$

$\Rightarrow \frac{x_1+2}{2} = 1 \text{ and } \frac{y_1+6}{2} = 4$

$\Rightarrow x_1 + 2 = 2$ and $y_1 + 6 = 8$

$\Rightarrow x = 0$ and $y = 2$.

The point on directrix is (0, 2).

We know that equation of the straight line passing through point (x_1, y_1) and slope m is $y - y_1 = m(x - x_1)$.

$\Rightarrow y - 2 = \frac{-1}{2}(x - 0)$

$\Rightarrow 2(y - 2) = -1(x)$

$\Rightarrow 2y - 4 = -x$

$\Rightarrow x + 2y - 4 = 0$

∴The correct option is A

9. Question

If V and S are respectively the vertex and focus of the parabola $y^2 + 6y + 2x + 5 = 0$, then SV =

A. 2

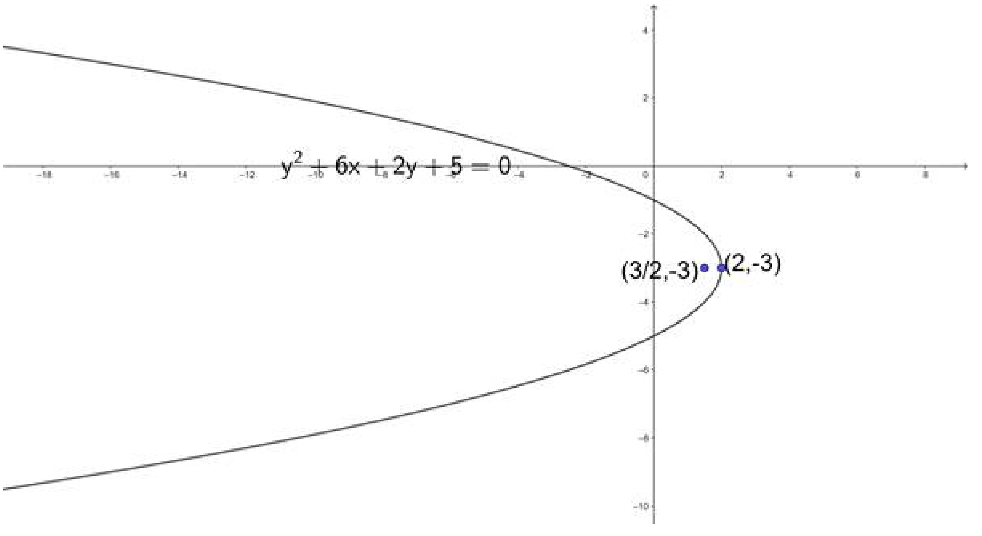
B. 1/2

C. 1

D. none of these

Answer

Given equation of the parabola is $y^2 + 6y + 2x + 5 = 0$



$\Rightarrow y^2 + 6y + 5 = - 2x$

$\Rightarrow y^2 + 6y + 9 = - 2x + 4$

$\Rightarrow (y + 3)^2 = - 2(x - 2)$

Comparing with standard form of parabola $(y - a)^2 = - 4b(x - c)$ we get,

$\Rightarrow 4b = 2$

$\Rightarrow b = \frac{2}{4}$

$\Rightarrow b = \frac{1}{2}$

We know that the distance between the vertex and the focus is b.

∴The distance between the vertex and the focus is $\frac{1}{2}$.

∴The correct option is B

10. Question

The directrix of the parabola $x^2 – 4x – 8y + 12 = 0$ is

A. y = 0

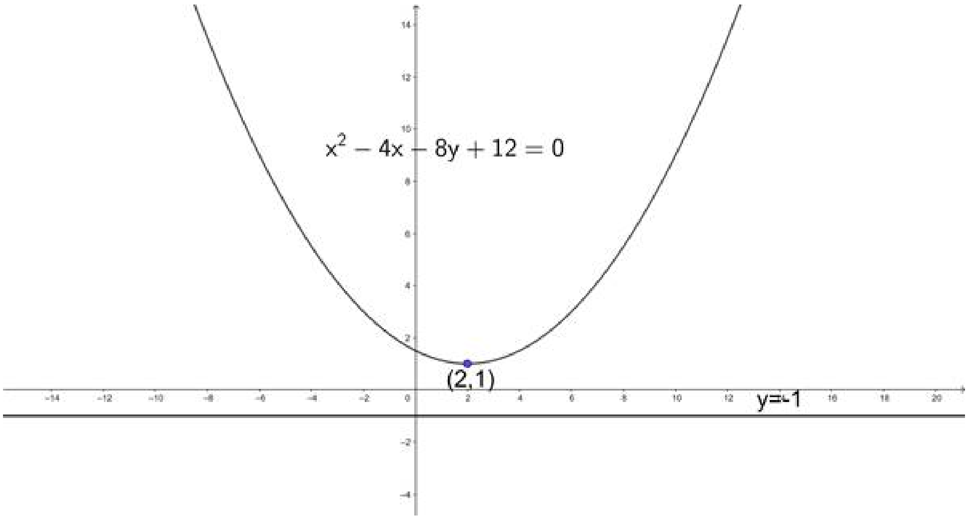
B. x = 1

C. y = - 1

D. x = - 1

Answer

Given equation of the parabola is $x^2 - 4x - 8y + 12 = 0$



⇒ $x^2 - 4x + 12 = 8y$

⇒ $x^2 - 4x + 4 = 8y - 8$

⇒ $(x - 2)^2 = 8(y - 1)$

Comparing with the standard form of parabola $(x - a)^2 = 4b(y - c)$ we get,

⇒ $4b = 8$

⇒ $b = 2$

⇒ The equation of the directrix is $y - c = - b$

⇒ Directrix is $y - 1 = - 2$

⇒ Directrix is $y = - 2 + 1$

⇒ Directrix is $y = - 1$

∴The correct option is C

11. Question

The equation of the parabola with focus (0, 0) and directrix $x + y = 4$ is

A. $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$

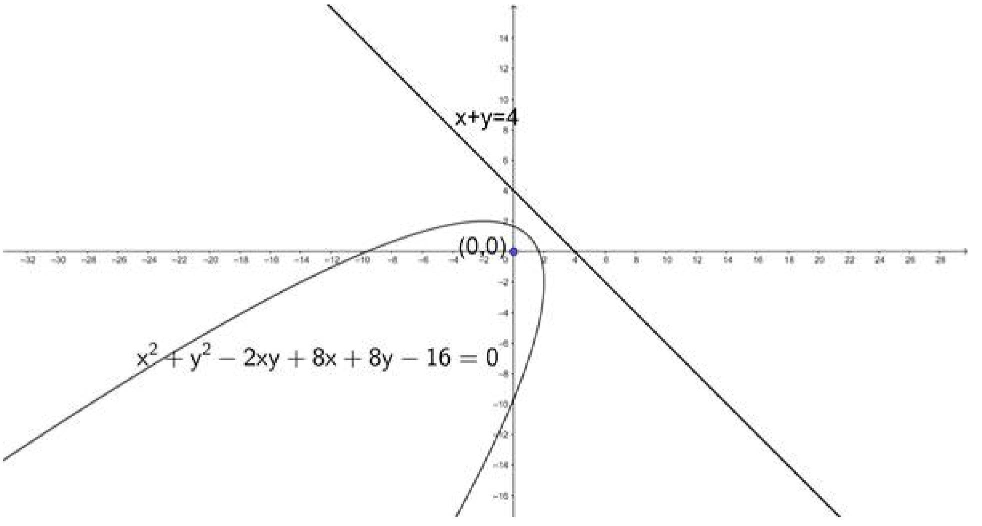
B. $x^2 + y^2 - 2xy + 8x + 8y = 0$

C. $x^2 + y^2 + 8x + 8y - 16 = 0$

D. $x^2 + y^2 + 8x + 8y - 16 = 0$

Answer

Given that we need to find the equation of the parabola with focus (0, 0) and directrix $x + y = 4$



Let us assume P(x, y) be any point on the parabola.

We know that the point on the parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 0)^2 + (y - 0)^2 = \left(\frac{|x + y - 4|}{\sqrt{1^2 + 1^2}}\right)^2$

⇒ $x^2 + y^2 = \frac{(|x + y - 4|)^2}{1 + 1}$

⇒ $2x^2 + 2y^2 = x^2 + y^2 + 16 + 2xy - 8x - 8y$

⇒ $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$

∴The correct option is A

12. Question

The line 2x – y + 4 = 0 cuts the parabola $y^2 = 8x$ in P and Q. The mid - point of PQ is

A. (1, 2)

B. (1, - 2)

C. (- 1, 2)

D. (- 1, - 2)

Answer

Given that the line 2x - y + 4 = 0 cuts the parabola $y^2 = 8x$ at P, Q. We need to find the midpoint of PQ.

Substituting y = 2x + 4 in the equation of parabola.

⇒ $(2x + 4)^2 = 8x$

⇒ $4x^2 + 16x + 16 = 8x$

⇒ $4x^2 + 8x + 16 = 0$

⇒ $x^2 + 2x + 4 = 0$

Let x_1, x_2 be the roots. Then, $x_1 + x_2 = - 2$

Now substituting $x = \frac{y^2}{8}$ in the equation of the line we get,

$\Rightarrow 2\left(\frac{y^2}{8}\right) - y + 4 = 0$

$\Rightarrow \frac{y^2}{4} - y + 4 = 0$

⇒ $y^2 - 4y + 16 = 0$

Let y_1, y_2 be the roots. Then, $y_1 + y_2 = 4$

Let us assume P be (x_1, y_1) and Q be (x_2, y_2) and R be the midpoint of PQ.

$\Rightarrow R = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\Rightarrow R = \left(\frac{-2}{2}, \frac{4}{2}\right)$

⇒ R = (- 1, 2)

∴The correct option is C

13. Question

In the parabola $y^2 = 4ax$, the length of the chord passing through the vertex and inclined to the axis at $\frac{\pi}{4}$ is

A. 4√2a

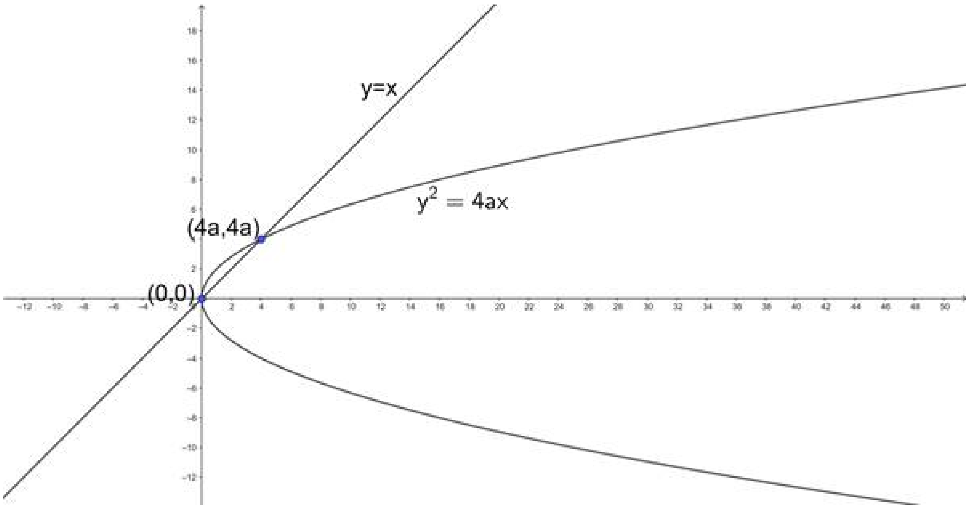
B. 2√2a

C. √2a

D. none of these

Answer

Given that we need to find the length of the chord of the parabola $y^2 = 4ax$ which passes through the vertex and is inclines to axis at $\frac{\pi}{4}$.



We know that the vertex and axis of the parabola $y^2 = 4ax$ is (0, 0) and y = 0(x - axis).

We know that the equation of the straight line passing through the origin and inclines to the x - axis at an angle θ is y = tanθx.

$\Rightarrow y = \tan\left(\frac{\pi}{4}\right)x$

⇒ y = 1.x

⇒ y = x.

The equation of the chord is y = x.

Substituting y = x in the equation of parabola.

$\Rightarrow x^2 = 4ax$

$\Rightarrow x = 4a.$

$\Rightarrow y = x = 4a$

The chord passes through the points (0, 0) and (4a, 4a).

We know that the distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow l = \sqrt{(0 - 4a)^2 + (0 - 4a)^2}$

$\Rightarrow l = \sqrt{16a^2 + 16a^2}$

$\Rightarrow l = \sqrt{32a^2}$

$\Rightarrow l = 4\sqrt{2}\, a$

∴The correct option is A

14. Question

The equation $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$ represents

A. A circle

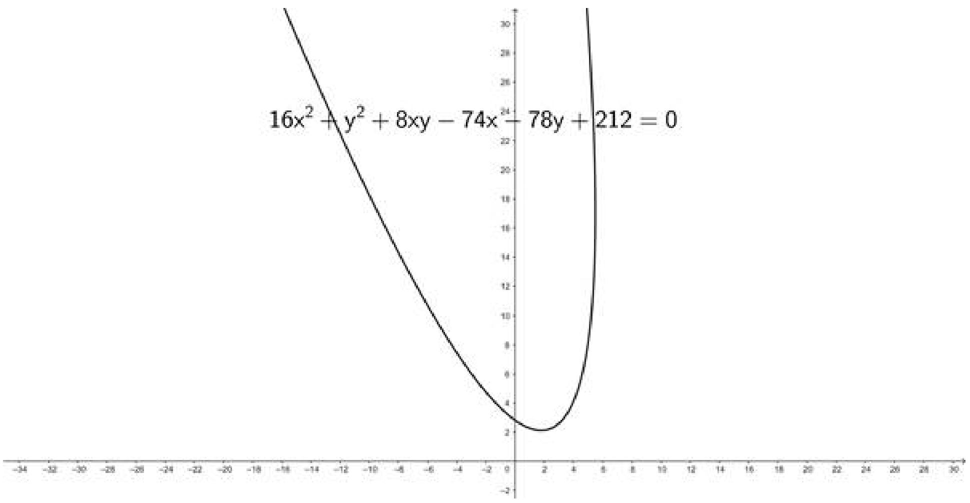
B. A parabola

C. An ellipse

D. A hyperbola

Answer

Given equation is $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$



We know that for $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is a parabola if $h^2 = ab$ and $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$

Here a = 16, b = 1, h = 4, g = - 37, f = - 39, c = 212.

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = (16)(1)(212) + (2)(-39)(-37)(4) - (16)(-39)^2 - (1)(-37)^2 - (212)(4)^2$

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 3392 + 11544 - 24336 - 1369 - 3392$

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = -14161$

$\Rightarrow h^2 = (4)^2$

⇒ $h^2 = 16$

⇒ $h^2 = (16)(1)$

⇒ $h^2 = ab$

The given curve is parabola.

∴The correct option is B

15. Question

The length of the latus - rectum of the parabola $y^2 + 8x - 2y + 17 = 0$ is

A. 2

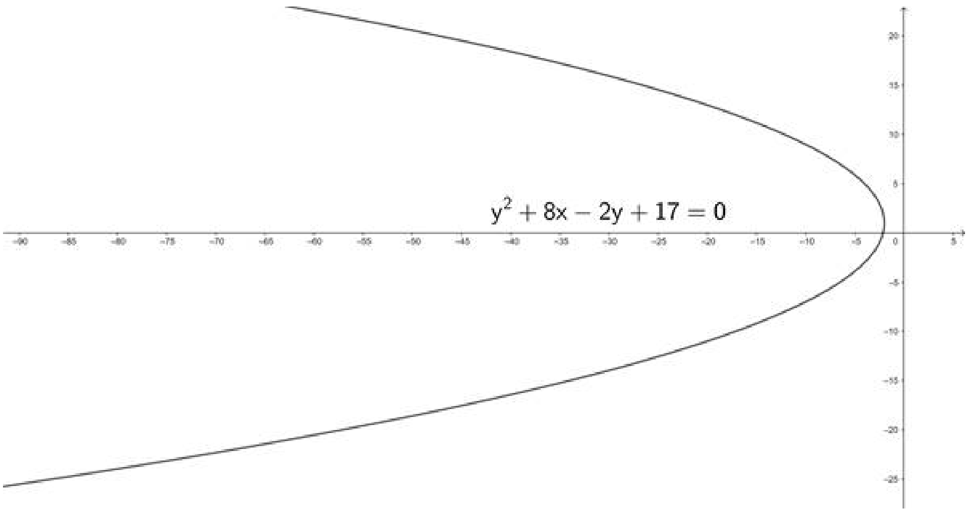
B. 4

C. 8

D. 16

Answer

Given equation of the parabola is $y^2 + 8x - 2y + 17 = 0$



⇒ $y^2 - 2y + 17 = - 8x$

⇒ $y^2 - 2y + 1 = - 8x - 16$

⇒ $(y - 1)^2 = - 8(x + 2)$

Comparing with standard form of parabola $(y - a)^2 = - 4b(x - c)$ we get,

⇒ $4b = 8$

We know that the that the length of the latus rectum is 4b.

∴The correct option is C

16. Question

The vertex of the parabola $x^2 + 8x + 12y + 4 = 0$ is

A. (- 4, 1)

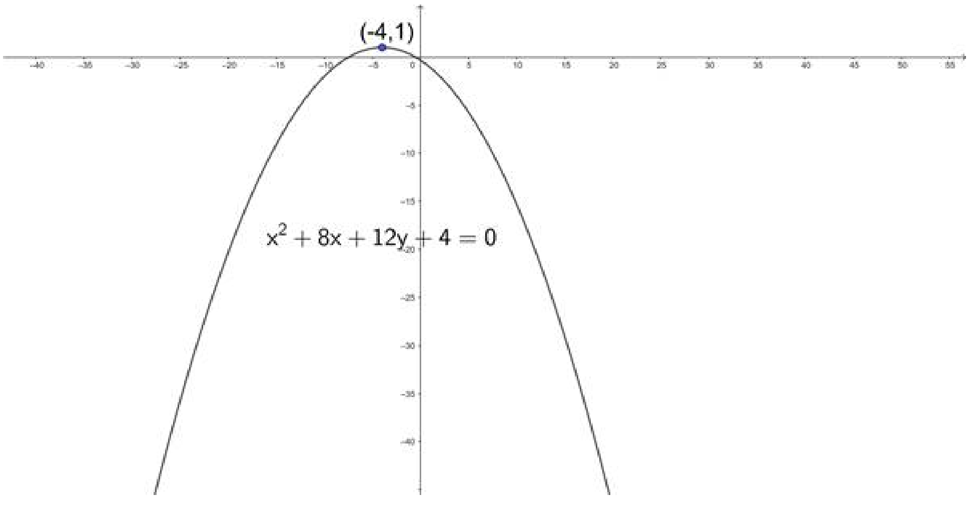
B. (4, - 1)

C. (- 4, - 1)

D. (4, 1)

Answer

Given equation of the parabola is $x^2 + 8x + 12y + 4 = 0$



$\Rightarrow x^2 + 8x + 4 = - 12y$

$\Rightarrow x^2 + 8x + 16 = - 12y + 12$

$\Rightarrow (x + 4)^2 = - 12(y - 1)$

Comparing with standard form of parabola $(x - a)^2 = - 4b(y - c)$ we get,

$\Rightarrow$ vertex = (a, c) = (- 4, 1)

∴The correct option is A

17. Question

The vertex of the parabola $(y – 2)^2 = 16(x – 1)$ is

A. (1, 2)

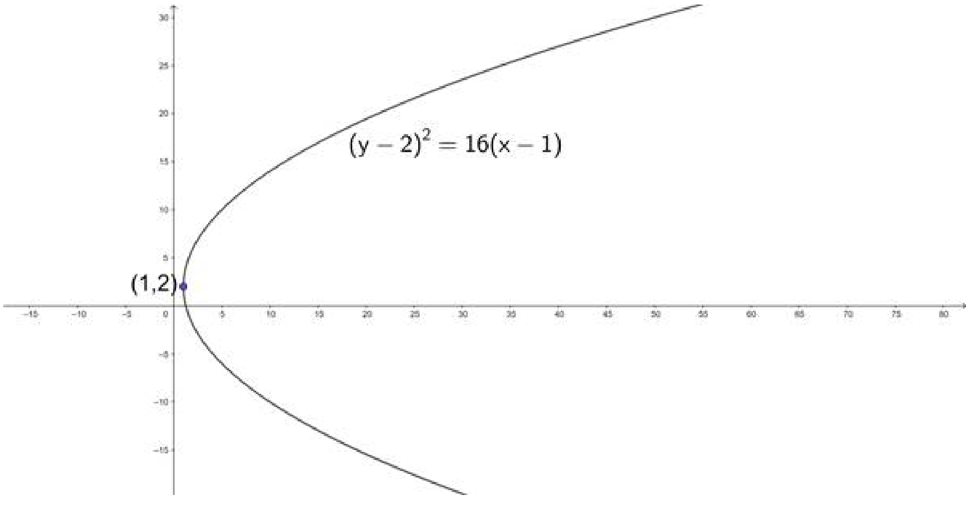
B. (- 1, 2)

C. (1, - 2)

D. (2, 1)

Answer

Given equation of the parabola is $(y - 2)^2 = 16(x - 1)$



Comparing with standard form of parabola $(y - a)^2 = - 4b(x - c)$ we get,

$\Rightarrow$ vertex = (c, a) = (1, 2)

∴The correct option is A

18. Question

The length of the latus - rectum of the parabola $4y^2 + 2x - 20y + 17 = 0$ is

A. 3

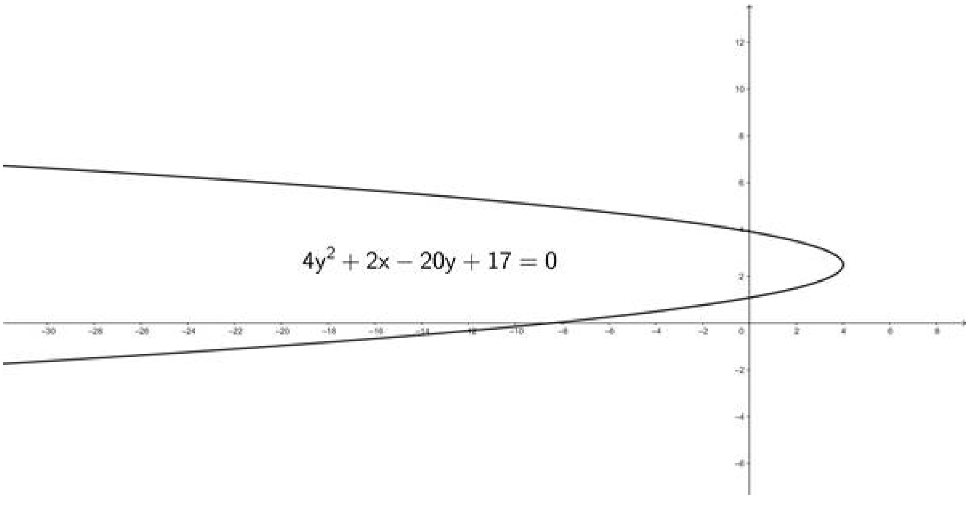
B. 6

C. 1/2

D. 9

Answer

Given equation of the parabola is $4y^2 + 2x - 20y + 17 = 0$



$\Rightarrow 4y^2 - 20y + 17 = -2x$

$\Rightarrow y^2 - 5y + \frac{17}{4} = \frac{-2}{4}x$

$\Rightarrow y^2 - 5y + \frac{25}{4} = \left(-\frac{1}{2}\right)x + 2$

$\Rightarrow y^2 - 5y + \frac{25}{4} = \left(\frac{-1}{2}\right)(x - 4)$

Comparing with standard form of parabola $(y - a)^2 = -4b(x - c)$ we get,

$\Rightarrow 4b = \frac{1}{2}$

We know that the that the length of the latus rectum is 4b.

∴The correct option is C

19. Question

The length of the latus - rectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is

A. 4

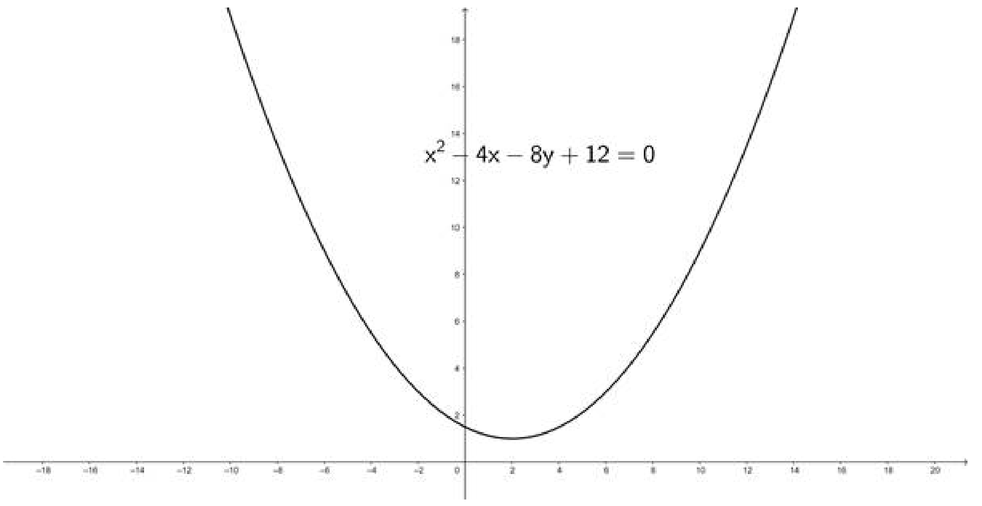
B. 6

C. 8

D. 10

Answer

Given equation of the parabola is $x^2 - 4x - 8y + 12 = 0$



$\Rightarrow x^2 - 4y + 12 = 8y$

$\Rightarrow x^2 - 4x + 4 = 8y - 8$

$\Rightarrow (x - 2)^2 = 8(y - 1)$

Comparing with standard form of parabola $(x - a)^2 = 4b(y - c)$ we get,

$\Rightarrow 4b = 8$

We know that the that the length of the latus rectum is 4b.

∴The correct option is C

20. Question

The focus of the parabola $y = 2x^2 + x$ is

A. (0, 0)

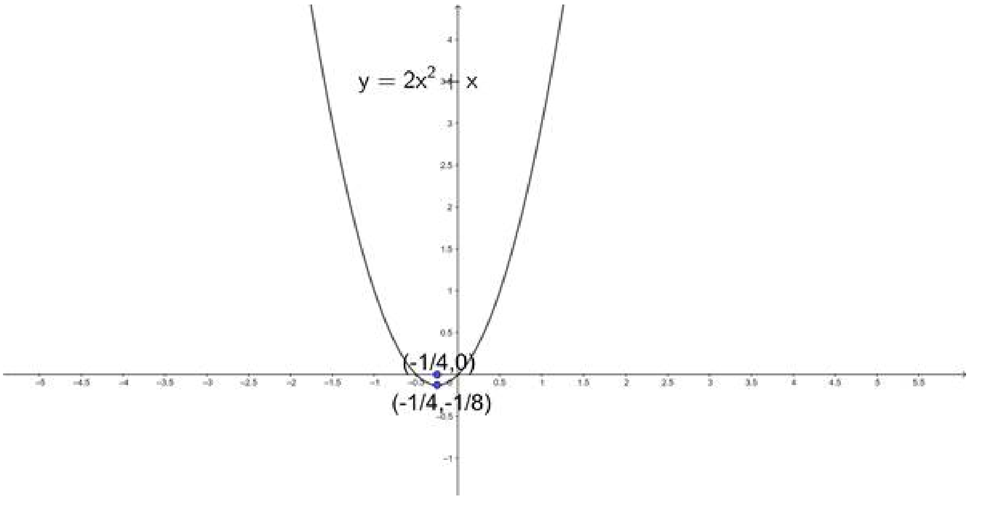
B. $\left(\frac{1}{2}, \frac{1}{4}\right)$

C. $\left(\frac{-1}{4}, 0\right)$

D. $\left(\frac{-1}{4}, \frac{1}{8}\right)$

Answer

Given equation of the parabola is $y = 2x^2 + x$



$\Rightarrow x^2 + \frac{x}{2} = \frac{y}{2}$

$\Rightarrow x^2 + \frac{x}{2} + \frac{1}{16} = \frac{y}{2} + \frac{1}{16}$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{2}\left(y + \frac{1}{8}\right)$

Comparing with standard form of parabola $(x - a)^2 = 4b(y - c)$ we get,

$\Rightarrow 4b = \frac{1}{2}$

$\Rightarrow b = \frac{1}{8}$

$\Rightarrow$ Focus = (a, b + c) = $\left(\frac{-1}{4}, \frac{1}{8} - \frac{1}{8}\right) = \left(\frac{-1}{4}, 0\right)$

∴The correct option is C

21. Question

Which of the following points lie on the parabola $x^2 = 4ay$?

A. $x = at^2$, $y = 2at$

B. $x = 2at$, $y = at^2$

C. $x = at^2$, $y = at$

D. $x = 2at$, $y = at$

Answer

Given that the equation of the parabola is $x^2 = 4ay$.

We know the parametric equation of the point on the parabola is $(2at, at^2)$

∴The correct option is B

22. Question

The equation of the parabola whose focus is (1, - 1) and the directrix is x + y + 7 = 0 is

A. $x^2 + y^2 - 2xy - 18x - 10y = 0$

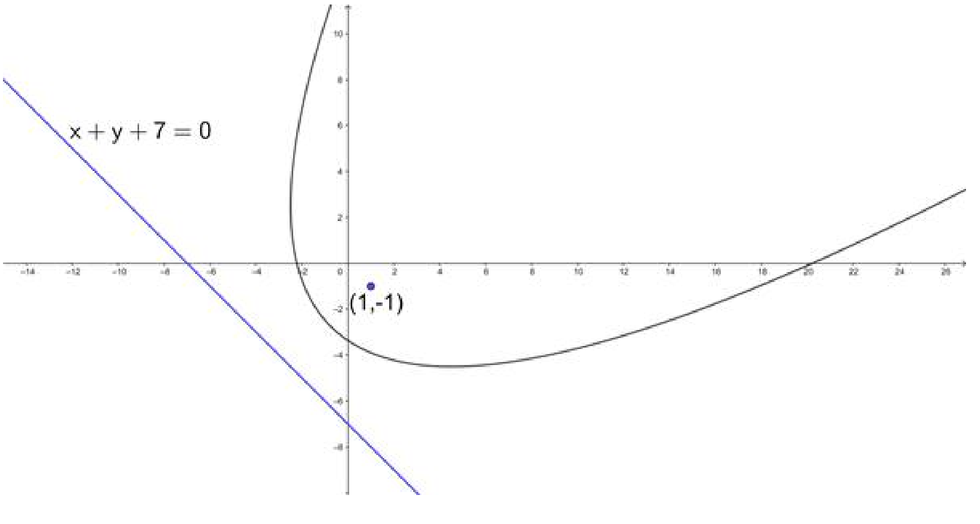
B. $x^2 - 18x - 10y - 45 = 0$

C. $x^2 + y^2 - 18x - 10y - 45 = 0$

D. $x^2 + y^2 - 2xy - 18x - 10y - 45 = 0$

Answer

Given that we need to find the equation of the parabola with focus (1, - 1) and directrix x + y + 7 = 0



Let us assume P(x, y) be any point on the parabola.

We know that the point on parabola is equidistant from focus and directrix.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

We know that the perpendicular distance from a point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

⇒ SP = PM

⇒ $SP^2 = PM^2$

⇒ $(x - 1)^2 + (y - (-1))^2 = \left(\frac{|x + y + 7|}{\sqrt{1^2 + 1^2}}\right)^2$

⇒ $x^2 - 2x + 1 + y^2 + 2y + 1 = \frac{(|x + y + 7|)^2}{1 + 1}$

⇒ $2x^2 + 2y^2 - 4x + 4y + 4 = x^2 + y^2 + 49 + 2xy + 14x + 14y$

⇒ $x^2 + y^2 - 2xy - 18x - 10y - 45 = 0$

∴The correct option is D